DOCUMENT RESUME

ED 026 849 24 EM 006 685

By-Naddor, Eliezer
Inventory Systems Laboratory. Final Report.
Johns Hopkins Univ., Baltimore, Md.
Spons Agency-Office of Education (DHEW), Washington, D.C. Bureau of Research.
Bureau No-BR-7-C-015
Pub Date Jan 68
Contract-OEC-7-070015-3111
Note-136p.
EDRS Price MF-$0.75 HC-$6.90
Descriptors-Business Cycles, Business Education, *Computer Assisted Instruction, *Computer Based Laboratories, *Computer Programs, Cost Effectiveness, Input Output Analysis, Instructional Programs, Laboratory Manuals, Laboratory Procedures, Laboratory Training, Models, *Sequential Programs, *Systems Analysis, Systems Development, Time Sharing
Identifiers-*Inventory Systems Laboratories

Four computer programs to aid students in understanding inventory systems, constructing mathematical inventory models, and developing optimal decision rules are presented. The program series allows a user to set input levels, simulates the behavior of major variables in inventory systems, and provides performance measures as output. Inventory Systems Lab (ISL)1 deals with carrying, shortage, and replenishment costs. The user selects three parameters: unit costs, interim demand, and planning horizon. He then must decide when and in what quantities replenishments are to be made. The program enables him to observe effects of changing parameters and/or replenishments on overall costs. ISL-2 and 3 introduce the user to factor optimization. For any reorder point and lot size set, interim system behavior and average costs are available. The user may then build a model for long-run system behavior, formulate optimization decision rules, and have the program test his results. In ISL-4 a user faces a system with a variety of properties and policies including lost sales, prescribed variable demand, and fixed inventory policies for which he carries out model building and optimization exercises. The program series is considered flexible and effective as a heuristic aid. Fortran IV listings are included. (SS)

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FINAL REPORT

Project No. 7-C-015

Contract No. OEC-7-070015-3111

INVENTORY SYSTEMS LABORATORY

January 1968

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FINAL REPORT

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INVENTORY SYSTEMS LABORATORY

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Baltimore, Maryland 21218

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Acknowledgments

I am indebted to the following students at The Johns Hopkins University for their assistance in preparing and writing of programs and manuals. Ole Braaten - Program and Manual ISL-1; Richard Kessinger - FORTRAN version of ISL-4; Israel Pressman - Program ISL-3; Richard Sacher - Program and Manual ISL-2, FORTRAN version of ISL-1, ISL-2, and ISL-3; Jon Weston - FORTRAN version of ISL-4.

I am also indebted to Mrs. Helen Macaulay for the typing of this report and Manuals ISL-1, ISL-2, ISL-3, and ISL-4.

E.N.

Summary

A computer laboratory is presented for assisting students in understanding inventory systems, constructing mathematical inventory models, and finding optimal decision rules. Four computer programs and the principles guiding their preparation are discussed:

ISL-1: Balancing Carrying, Shortage, and Replenishing Costs

ISL-2: The Deterministic Reorder Point--Lot Size System

ISL-3: The Probabilistic Reorder Point--Lot Size System

ISL-4: A General Inventory Systems Simulation

Four manuals describing the use of the programs are included. Each manual specifies how the unit costs, the demands for inventory, and a variety of policy decisions are presented to the computer. It gives illustrations of actual inputs and outputs, and also suggestions on model construction and/or optimization. Observations are made on the methodology of production of computer laboratory manuals.

The use of the laboratory by students is discussed. Observations are made on adapting the programs to any computer system, on testing the laboratory, and on extension of the work to other fields.

1. Introduction

1.1 Inventory systems is a field in which carrying, shortage, and replenishment costs can be controlled by making appropriate decisions (see references [1], [2], [3], [5], and [6]). This field has a wide range of theoretical and practical applications. It is particularly suitable for the study of model building and the derivation of optimal decision rules. Students of inventory systems can extend their learning experience to other fields in Operations Research, Management Science, Industrial Engineering, Economics, Business Administration, etc.

1.2 Computer time sharing systems can now be used economically as an aid in the teaching process. They seem to be particularly useful in simulating an ideal laboratory environment for the study of inventory systems. This report describes computer programs and manuals for such an inventory systems laboratory.

1.3 The laboratory has been designed to give the student three major options for each inventory system under study:

A. Simulation of inventory fluctuation
B. Long term averages
C. Table of total costs

He can use the options to check whether he understands the inventory system (Option A), whether the model he has constructed seems to be correct (Option B), and whether he has found reasonably good decision rules (Option C).

2. Inventory Systems Computer Programs

2.1 Inventory systems are characterized according to such factors as costs (carrying, shortage, and replenishing), demands (deterministic or probabilistic), leadtime (zero or non-zero), and policy (when to order and how much). By varying these and other factors, the number of different inventory systems that can be constructed is practically infinite. This report deals with only four inventory systems. Each is described in detail in an appropriate manual in the appendices.

2.2 The first system (ISL-1) is intended mainly to give the student an appreciation for the need to balance carrying, shortage, and replenishing costs. In the second system (ISL-2) demand is constant and leadtime is zero. This is an excellent system for learning how to build a model and how to determine optimal decision rules.

The third system (ISL-3), which is an extension of ISL-2, allows demand to be both deterministic and probabilistic. It is a relatively complex system for which it is rather difficult to construct a model. The problem of finding optimal decision rules is even more difficult.

The fourth system (ISL-4) is a rather general inventory system. It allows the study of systems with a variety of properties and policies. These include lost sales, probabilistic leadtime, prescribed variable demand, and the inventory policies (z,q), (z,Z), and (t,Z). Systems with some of these properties are known to be extremely difficult to analyze. The simulation option of Program ISL-4 is therefore particularly useful for the study of such systems.

2.3 A number of principles were used in preparing the programs for the four inventory systems.

A. The output from the computer will consist only of requests for data and instructions, and the execution of the instructions.

B. As many data and instructions as possible will be presented at one time.

C. A special feature for checking correct results is not necessary.

Principle A essentially means that the program will not provide text material of any sort. Such material should be in the manual. Principle B ensures concentrated decisions by the student and economic utilization of the computer. Principle C has been used in the belief that the student is the better judge of correct results.

2.4 The mathematical formulation of the programs is based on reference [5]. Program ISL-1 is essentially Example 1-0, pp. 4-7. Program ISL-2 is related to Section 5-1, pp. 79-84. Program ISL-3 is mostly based on Section 13.3, pp. 246-252. Most of the mathematical formulation of Program ISL-4 is new. Portions of the program are based on Section 12-3, pp. 217-224.

The main coding and implementation was done in the BASIC language [4] on the GE-265 Time Sharing System. At the end of each manual the full listing of the program is given. All illustrations in the manuals exemplify inputs and outputs on the GE-265 system.

The programs were also coded and implemented in the FORTRAN IV language on the IBM-7094 Batch Processing System. The listings of the programs are given in the Appendix E. The outputs from these programs, using the data following each listing, are almost identical with the corresponding outputs in the manuals.

3. Inventory Systems Laboratory Manuals

3.1 We consider the preparation of manuals to be a most important task in using computers in the teaching process. The manuals in Appendices A to D should be regarded only as a first attempt to exemplify the methodology which is involved.

3.2 Several principles have guided us in preparing the manuals.

A. The manual should only deal with the laboratory. Instruction on inventory systems should be given in the classroom and/or in a textbook.
B. The manual should be read before the laboratory. The student should come to the laboratory prepared with data for the computer.
C. The manual should contain numerous illustrations which provide examples of the precise inputs to the computer and actual copies of output from the computer.

3.3 Each of the manuals in Appendices A to D includes the following sections:

A. Presentation of data
B. Illustrations
C. Construction of model and/or optimization
D. Extensions and problems for solution
E. Listing of the BASIC program

Section A describes the parameters, the controllable variables, and the available output options. It gives the names of all these variables in the program. Actual examples of data and/or inputs are also given.

Section B provides illustrations of actual computer runs with data and/or inputs, and outputs.

In Section C a few suggestions are given on the construction of the model and/or on the method of finding optimal decision rules.

Section D discusses some of the limitations of the system and suggests possible extensions. All these extensions require changes in the computer program. The section also includes several problems for solution using the program as it is.

Section E gives the full listing of the computer program as it is coded in BASIC and stored in the GE-265 system. The listing is mainly intended for the teacher.

3.4 The actual production of a manual is a time consuming job which requires extensive planning. Much thought must be given to the illustrations, to ensure exemplification of all the features and shortcomings of the computer program.

We used the editing capabilities of the Time Sharing System for the production of portions of the manuals. It was hoped that the complete manuals could be stored in the computer so that changes could easily be made. However, our experience has shown that this method of production was rather time consuming and costly.

4. Use of the Laboratory by Students

4.1 Full use of the laboratory will be made at The Johns Hopkins University in the academic year 1968-69 in the undergraduate course "Elementary Inventory Systems." This course was not offered in 1967-68.

4.2 The laboratory was pretested by graduate and undergraduate students at Johns Hopkins. It was also used in the author's courses "Advanced Inventory Systems" in the Fall of 1967, on a number of occasions. This preliminary use seems to indicate that the laboratory is an effective teaching device. It was gratifying to see the smile on a student's face when the long term averages which he predicted were actually printed by the computer. It was interesting to watch another student who claimed to have found the optimal decision rules of a system. He used the option in the program which gave the costs in the neighborhood of his solution but to his surprise, the solution was not optimal. Eventually he found the correct solution and he also smiled.

4.3 The following observation can be made regarding the maximum benefit that can be derived from the laboratory.

A. Students should be assigned several problems for solution in advance. They should come to the laboratory with several sets of data and a full list of the results they expect from the computer.

B. Students should work in groups of, say, 3 to 5 students. Each group should be supervised by a laboratory assistant.

C. Whenever it is appropriate, students should submit a brief report documenting their work.

5. Conclusion

5.1 The inventory system laboratory described in this report is ready for use by students on the GE-265 Time Sharing System and on the IBM-7094 Batch Processing System. It can be adapted for use on any other computer system with relative ease.

5.2 Several principles have been suggested to guide the preparation of the computer programs and the manuals for the laboratory. These principles, as well as the laboratory as a whole, should now be subjected to detailed testing.

5.3 The methodology developed for the inventory systems laboratory can be readily applied to other fields. In particular, the following fields within Operations Research seem to be especially suitable: queueing theory, game theory, response surface analysis, stochastic allocation models, reliability theory, and replacement models.

6. References

[1] Buchan, Joseph and Ernest Koenigsberg,"Scientific Inventory Management," Prentice-Hall, Englewood Cliffs, N.J., 1963.

[2] Hadley, G., and T. M. Whitin, "Analysis of Inventory Systems,"Prentice-Hall, Englewood Cliffs, N.J., 1963.

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[6] Starr, Martin, and David W. Miller, "Inventory Control: Theory and Practice," Prentice-Hall, Englewood Cliffs, N.J., 1962.

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INVENTORY SYSTEMS LABORATORY ONE (ISL-1)

Manual 1A

(Revision A)

by

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September 1967

Supported by the Office of Education
Contract OEC2-7-070015-3111

1. Introduction

The main purpose of the inventory system used in this laboratory (System 1) is to introduce the student to the balancing of carrying costs, shortage costs, and replenishment costs, by trial and error methods. In other laboratories (ISL-2, ISL-3, ISL-4) the inventory systems can be used for construction of mathematical models and for evaluation of optimal decision rules.

2. Properties of System 1

2.1 Demands

The planning horizon is composed of N periods. The demand D(I) at the beginning of period I is known. The student supplies the values of N, D(1),...,D(N).

2.2 Unit Costs

The cost of carrying inventory is C1 per unit per period. The cost of shortage is C2 per unit per period. The cost of replenishing is C3 per replenishment. The student supplies the values of C1, C2, and C3.

2.3 Decisions

The program allows for two types of decisions:

A. The initial inventory which is available at the beginning of the first period, B(1). There is no replenishing cost associated with this amount.

B. When should replenishments be made and in what quantities. Up to N replenishments may be specified. A cost of C3 is associated with each replenishment. The decision variables are I(1), R(I(1)),...,I(M), R(I(M)), where I designates a period at the beginning of which there is a replenishment of R(I) and where M is the number of replenishments.

2.4 Constraints

It is assumed that the demands during the planning horizon are exactly met by replenishments. That is,

$$D(1) + \ldots + D(N) = R(I(1)) + \ldots + R(I(M))$$

Thus the initial inventory B(1) is equal to the amount on hand at the end of the horizon.

2.5 Objectives

For the given demands and unit costs the student must determine by trial and error the decisions which will give a minimum total cost of the inventory system.

3. Presentation of Data

3.1 Periods: N

The planning horizon is composed of a specified number of periods. These periods may be days, weeks, months, etc. The number of periods is given in a data statement:

```
910 DATA < N >
```

For example, for a planning horizon of 12 months the data statement is:

```
910 DATA 12
```

3.2 Demands: D(1),...,D(N)

In each period I there may arise a demand of D(I). The demand is given in a data statement:

```
920 DATA < D(1),...,D(N) >
```

For example, the demand in tons for the 12 month horizon would appear as:

```
920 DATA 10,20,20,30,20,30,0,0,40,30,20,20.
```

(Note that for each period there must correspond a number designating the demand for that period, even if that demand happens to be zero.)

3.3 Costs: C1, C2, C3

The carrying cost, C1, is measured in dollars per unit per period. For example: $.20 per ton per month. The shortage cost, C2, is measured in dollars per unit per period. For example: $5.00 per ton per month. The replenishing cost, C3, is measured in dollars. For example: $10.00 for each replenishment. The inventory costs are placed in a data statement, thus:

```
930 DATA < C1,C2,C3 >
```

For example,

```
930 DATA .20, 5.00, 10.00
```

3.4 Initial Inventory: B(1)

The program permits the inclusion of any initial inventory, B(1), left over from a previous planning horizon. This information is given in a data statement:

```
940 DATA < B(1) >
```

For example, for an initial inventory of 10 tons the data statement will be:

```
940 DATA 10
```

3.5 Replenishments: When - I, How much - R(I)

As with the initial inventory, the program also permits control of replenishments. One has to furnish M periods, I(1) to I(M), and with each period, I, a corresponding replenishment, R(I). This appears in a data statement as:

```
950 DATA < I(1), R(I(1)),...,I(M), R(I(M)) >
```

For example, suppose N = 12 months, and replenishments are desired at the beginning of the months: 2,4,9, and 11 with the quantities: 40, 80, 70, and 50 respectively. The data statement would then appear as:

```
950 DATA 2,40,4,80,9,70,11,50.
```

(When additional lines are needed to accommodate all the replenishment information, the lines from 951 through 959 may also be used.)

3.6 Summary

Five data lines are needed. The first three constitute the parameters of the inventory problem: the length of the planning horizon, the demands, and the unit costs. The last two lines are the controllable variables: the amount on hand at the beginning of the planning horizon and the replenishments during the horizon. The general form of the data statement is:

```
910 DATA < N >
920 DATA < D(1),...,D(N) >
930 DATA < C1,C2,C3 >
940 DATA < B(1) >
950 DATA < I(1), R(I(1)),...,I(M), R(I(M)) >
```

For example:

```
910 DATA 12
920 DATA 10,20,20,30,20,30,0,0,40,30,20,20
930 DATA .20,5.00,10.00
940 DATA 10
950 DATA 2,40,4,80,9,70,11,50
```

In Section 5.2 (p.12) the printouts resulting from these data are given.

4. Examples

Three examples are given in this section. In all the examples the planning horizon is composed of 12 months. On January 1 the demand is 10 tons, and at the beginning of the other 11 months the demands are: 20,20,30,20,30,0,0,40,30,20, and 20. The total annual demand is thus 240 tons.

The unit costs are also the same in all three examples. The carrying cost is $0.20 per ton per month, the shortage cost is $5.00 per ton per month, and the replenishing cost is $10.00 per replenishment.

In all the examples in this section it is assumed that replenishments occur at the same interval of time and that the quantity replenished is constant.

4.1 Example 1

Suppose the initial inventory on January 1 is zero and suppose that 4 replenishments of 60 tons each are scheduled for January 1, April 1, July 1, and October 1. The data and results would be:

```
910 DATA 12
920 DATA 10,20,20,30,20,30,0,0,40,30,20,20
930 DATA .20,5.00,10.00
940 DATA 0
950 DATA 1,60,4,60,7,60,10,60

RUN
WAIT.


ISL-1       11:32        08/04/67

PERIOD          BEGIN         REPLENISHMENT       DEMAND          END

 1               0                60              10              50
 2               50               0               20              30
 3               30               0               20              10
 4               10               60              30              40
 5               40               0               20              20
 6               20               0               30             -10
 7              -10               60              0               50
 8               50               0               0               50
 9               50               0               40              10
 10              10               60              30              40
 11              40               0               20              20
 12              20               0               20              0
```

```
                 CARRYING       SHORTAGE         REPLENISHMENT   TOTAL

AVERAGE          26.6667        .833333          .333333
UNIT COST        .2             5                10
COST/PERIOD      5.33333        4.16667          3.33333         12.8333

TOTAL COST       64.            50.              40.             154.

TIME:    1 SECS.
```

These results are self explanatory. The average amount carried is the average of the amounts in the last column, with the -10 replaced by 0. The average shortage is 10/12 = 0.83333. Since there were 4 replenishments, the average number of replenishments per month is 4/12 = 0.333333.

4.2 Example 2

Suppose the initial inventory on January 1 is again zero, but 6 replenishments of 40 tons each are scheduled. The data and results are now:

```
910 DATA 12
920 DATA 10,20,20,30,20,30,0,0,40,30,20,20
930 DATA .20,5.00,10.00
940 DATA 0
950 DATA 1,40,3,40,5,40,7,40,9,40,11,40

RUN
WAIT.
```

```
ISL-1       11:43          08/04/67

PERIOD             BEGIN         REPLENISHMENT        DEMAND          END

  1                 0                  40               10             30
  2                30                   0               20             10
  3                10                  40               20             30
  4                30                   0               30              0
  5                 0                  40               20             20
  6                20                   0               30            -10
  7               -10                  40                0             30
  8                30                   0                0             30
  9                30                  40               40             30
 10                30                   0               30              0
 11                 0                  40               20             20
 12                20                   0               20              0

                  CARRYING       SHORTAGE          REPLENISHMENT    TOTAL

AVERAGE           16.6667          .833333          .5
UNIT COST         .2               5                10
COST/PERIOD       3.33333          4.16667          5                12.5

TOTAL COST        40.              50.              60               150.


TIME:    1 SECS.
```

Example 2 illustrates how costs can be changed by a suitable change in replenishments. In Example 1 the total cost is $154.00 per year, whereas in Example 2 it is $150.00. The student should note that the increased replenishment costs were more than compensated for by the decreased carrying costs.

4.3 Example 3

Suppose now that replenishments are the same as in example 2 but that the initial inventory on January 1 is 10 tons. The data and the results are now:

```
910 DATA 12
920 DATA 10,20,20,30,20,30,0,0,40,30,20,20
930 DATA .20,5.00,10.00
940 DATA 10
950 DATA 1,40,3,40,5,40,7,40,9,40,11,40

RUN
WAIT.


ISL-1      11:47          08/04/67

PERIOD          BEGIN       REPLENISHMENT      DEMAND          END

 1              10              40             10              40
 2              40              0              20              20
 3              20              40             20              40
 4              40              0              30              10
 5              10              40             20              30
 6              30              0              30              0
 7              0               40             0               40
 8              40              0              0               40
 9              40              40             40              40
 10             40              0              30              10
 11             10              40             20              30
 12             30              0              20              10

                CARRYING      SHORTAGE        REPLENISHMENT   TOTAL

AVERAGE         25.8333         0              .5
UNIT COST       .2              5              10
COST/PERIOD     5.16667         0              5               10.1667

TOTAL COST      62.             0              60              122.

TIME:    1 SECS.
```

4.4 Summary

The costs in the three examples can now be summarized.

EXAMPLE	CARRYING	SHORTAGE	REPLENISHING	TOTAL
1	64	50	40	154
2	40	50	60	150
3	62	0	60	122

We note that by appropriate decisions any one of the three types of costs can be increased (or decreased). This usually results in a decrease (or an increase) in some other cost. The problem of finding the best decision is discussed in the next section.

In passing one should note that in every example the annual replenishments equalled 240 tons - the total annual demand. If the totals were not equal, say, if one replenishment of 230 tons is scheduled, then the following result will be printed:

```
910 DATA 12
920 DATA 10,20,20,30,20,30,0,0,40,30,20,20
930 DATA .20,5.00,10.00
940 DATA 10
950 DATA 1,230

RUN
WAIT.

ISL-1      11:57          03/04/67

REPLENISHMENTS ARE NOT EQUAL DEMANDS
CHECK LINES 920 AND 950

TIME:   1 SECS.
```

5. Optimization

Analytic methods of optimization are discussed in other manuals. The student should use program ISL-1 to find an optimal decision by trial and error. The examples in this section illustrate a trial and error approach.

5.1 Example 4

A careful study of the results of Example 3 indicates that inventories can be drastically reduced if replenishments are not constant but are equal to the corresponding 2 months' demands. Thus, for no initial inventory one can get:

```
910 DATA 12
920 DATA 10,20,20,30,20,30,0,0,40,30,20,20
930 DATA .20,5.00,10.00
940 DATA 0
950 DATA 1,30,3,50,5,50,9,70,11,40

RUN
WAIT.


ISL-1       12:02            03/04/67

PERIOD            BEGIN         REPLENISHMENT        DEMAND           END

 1                 0                30                10               20
 2                 20               0                 20               0
 3                 0                50                20               30
 4                 30               0                 30               0
 5                 0                50                20               30
 6                 30               0                 30               0
 7                 0                0                 0                0
 8                 0                0                 0                0
 9                 0                70                40               30
 10                30               0                 30               0
 11                0                40                20               20
 12                20               0                 20               0

                  CARRYING       SHORTAGE           REPLENISHMENT    TOTAL

AVERAGE            10.8333          0                .416667
UNIT COST          .2               5                10
COST/PERIOD        2.16667          0                4.16667          6.33333

TOTAL COST         26.              0                50.              76.

TIME:    1 SECS.
```

5.2 Example 5

Further trial and error attempts may eventually lead to the following results which seem to be optimal:

```
910 DATA 12
920 DATA 10,20,20,30,20,30,0,0,40,30,20,20
930 DATA .20,5.00,10.00
940 DATA 10
950 DATA 2,40,4,80,9,70,11,50

RUN
WAIT.


ISL-1      12:09          03/04/67

PERIOD          BEGIN         REPLENISHMENT       DEMAND          END

 1               10              0                 10              0
 2               0               40                20              20
 3               20              0                 20              0
 4               0               80                30              50
 5               50              0                 20              30
 6               30              0                 30              0
 7               0               0                 0               0
 8               0               0                 0               0
 9               0               70                40              30
 10              30              0                 30              0
 11              0               50                20              30
 12              30              0                 20              10


                CARRYING      SHORTAGE          REPLENISHMENT   TOTAL

AVERAGE         14.1667          0                .333333
UNIT COST       .2               5                10
COST/PERIOD     2.83333          0                3.33333         6.16667

TOTAL COST      34.              0                40.             74.

TIME:    1 SECS.
```

5.3 Example 6

As another example consider an inventory system with a planning horizon of 7 days. Each weekday there is a demand for 5 box cars. On Saturday there is a demand for 10 cars. There is no demand on Sundays. The carrying cost is $10.00 per car per day. The shortage cost is $20.00 per car per day. The replenishing cost is $400.00 per replenishment. What is an optimal replenishing policy?

One can first attempt to balance carrying costs and replenishing costs while avoiding shortages. Suppose that only one replenishment is made per week. Then 35 cars would have to be ordered, say, on Monday. The carrying cost would then be $1000 per week and the replenishing cost would be $400 per week for a total of $1400 per week.

For two replenishments per week, say 20 cars on Monday and 15 cars on Thursday, the carrying cost is $400 per week. The replenishing cost is $800 per week so that the total cost is $1200 per week.

For three replenishments per week the replenishing cost alone is $1200 per week. Hence, if shortages are to be avoided the optimal solution seems to be:

```
910 DATA 7
920 DATA 5,5,5,5,5,10,0
930 DATA 10,20,400
940 DATA 0
950 DATA 1,20,5,15

RUN
WAIT.
```

```
ISL-1       14:03              03/04/67

PERIOD              BEGIN         REPLENISHMENT         DEMAND           END

 1                   0                20                 5                15
 2                   15               0                  5                10
 3                   10               0                  5                5
 4                   5                0                  5                0
 5                   0                15                 5                10
 6                   10               0                  10               0
 7                   0                0                  0                0

                    CARRYING       SHORTAGE           REPLENISHMENT     TOTAL

AVERAGE              5.71429         0                  .285714
UNIT COST            10              20                 400
COST/PERIOD          57.1429         0                  114.286          171.429

TOTAL COST           400.            0                  800.             1200.

TIME:    1 SECS.
```

5.4 Example 7

In Example 6 no shortages were allowed. One can now consider shortages as follows:

Suppose that instead of two replenishments of 20 and 15 cars one chooses 19 and 16 respectfully. This will reduce inventories on Mondays, Tuesdays, and Wednesdays, but will cause a shortage on Thursday. The net effect, however, will be a saving of 3 x 10 - 20 = $10 per week. Similar considerations lead to the following solution which seems to be the optimal solution.

```
910 DATA 7
920 DATA 5,5,5,5,5,10,0
930 DATA 10,20,400
940 DATA 0
950 DATA 1,15,5,20

206
230

RUN
WAIT.


ISL-1      14:29           03/04/67



                   CARRYING     SHORTAGE        REPLENISHMENT   TOTAL

AVERAGE             3.57143       .714286        .285714
UNIT COST           10            20             400
COST/PERIOD         35.7143       14.2857        114.286        164.286

TOTAL COST          250.          100.           800.           1150.

TIME:    1 SECS.
```

Note that deletion of Lines 206 and 230 has eliminated the detailed printouts of the inventory fluctuations. The student may wish to use this option when he is interested only in the summary of the results.

6. Extensions and Problems for Solution

6.1 Extensions

System 1 is restricted in a number of ways. For example:

A. Leadtime is assumed to be zero.

B. Demands occur only at the beginning of each period.

C. The cost of shortages are measured in dollars per unit per period.

D. Demands are assumed to be known (deterministic).

It is possible to relax these restrictions. However, to do so one would need to change the Program ISL-1 (see Section 7). The student will find that changes in the properties A, B, and C above are relatively easy to handle. However, this is not the case with property D.

6.2 Problems for Solution

A. Find the solution of the system described in Section 4 (p. 5) when:

- A1. The carrying cost is \$2.00 per ton per month (instead of \$0.20 per ton per month).
- A2. The shortage cost is \$50.00 per ton per month (instead of \$5.00 per ton per month).
- A3. The replenishing cost is \$100.00 per replenishment (instead of \$10.00 per replenishment).

B. Find the solution of the system described in Example 6 (p. 13) when each unit cost is 10 times as large. (Solve these separate cases as Problem A above.)

C. Demand is known to cycle over a 14-day period: it is 10 on days 1,2, and 3; 15 on days 4 to 7; 5 on days 8,9, and 10; and 7 on days 11 to 14. The carrying cost is $1.00 per unit per day, the shortage cost is $1.00 per unit per day, and the replenishing cost is $10. What is an optimal replenishing policy?

D. Demand is uniform at the rate of 2400 points per year. The carrying cost is $0.56 per part per year. The ordering cost is $42 per order. No shortages are allowed. What is the optimal lot size?

E. The weekly demands in an inventory system during one year are 2,1,0,1,5,3.1,4,1,0,4,4,1,0,3,1,2,1,0,3,1,1,0,0,1,1,2,3,2,0,4,2,1,0,2,0,3,1,1, 1,2,2,3,0,1,1,1,1,3. The carrying cost is $1.00 per pound per week, the shortage cost is $10.00 per pound per week, the replenishing cost is $30.00 per replenishment. Find an optimal replenishing policy.

F. Solve the system of Section 4 (p. 5) when the monthly demands are equal to 20 tons each.

G. Solve Example 6 (p. 13) when the Saturday and Sunday demands are for 5 cars (instead of 10 and 0 cars).

7. Program ISL-1

```
01 REM ISL-1 FOR MANUAL 1
02 REM BY E.NADDOR AND O.BRAATEN, THE JOHNS HOPKINS UNIVERSITY
95 DIM B(60),R(60),D(60),E(60)
100 LET D = 0
105 LET R = 0
110 READ N
115 FOR I = 1 TO N
120 READ D(I)
122 LET D = D + D(I)
125 NEXT I
127 READ C1,C2,C3
128 READ B(1)
130 READ I
135 IF I=-99 THEN 150
137 READ R(I)
140 LET R=R + R(I)
142 GO TO 130
150 IF D = R THEN 200
155 PRINT "REPLENISHMENTS ARE NOT EQUAL DEMANDS"
160 PRINT "CHECK LINES 920 AND 950"
170 STOP
200 LET I1 = 0
202 LET I2 = 0
204 LET I3 = 0
206 PRINT "PERIOD", "BEGIN        REPLENISHMENT", "DEMAND", "END"
208 PRINT
210 FOR I = 1 TO N
220 LET E(I) = B(I) + R(I) - D(I)
230 PRINT I, B(I), R(I),D(I),E(I)
240 IF E(I)>=0 THEN 250
245 LET I2 = I2 - E(I)
247 GO TO 270
250 LET I1 = I1 + E(I)
270 IF R(I)> 0 THEN 280
275 GO TO 285
280 LET I3 = I3 + 1
285 LET B(I + 1) = E(I)
290 NEXT I
300 LET I1 = I1/N
305 LET I2 = I2/N
310 LET I3=I3/N
315 PRINT
316 PRINT
320 PRINT " ", "CARRYING     SHORTAGE          REPLENISHMENT", "TOTAL"
325 PRINT
330 PRINT "AVERAGE", I1,I2,I3
335 PRINT "UNIT COST", C1,C2,C3
340 PRINT "COST/PERIOD",I1*C1, I2*C2, I3*C3, I1*C1+I2*C2+I3*C3
345 PRINT
350 PRINT "TOTAL COST", I1*C1*N, I2*C2*N, I3*C3*N,
351 PRINT I1*C1*N+I2*C2*N+I3*C3*N
910 DATA 12
920 DATA 10,20,20,30,20,30,0,0,40,30,20,20
930 DATA .20,5.00,10.00
940 DATA 10
950 DATA 2,40,4,30,9,70,11,50
998 DATA -99
999 END
```

INVENTORY SYSTEMS LABORATORY TWO (ISL-2)

Manual 2

by

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and

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December 1967

Supported by the Office of Education
Contract OEC2-7-070015-3111

1. Introduction

This laboratory deals with a deterministic reorder point - lot size system. The system is an extension of ISL-1 and forms a basis for two other laboratories: ISL-3 and ISL-4. Program ISL-2 has 3 distinct uses:

A. For any reorder point and lot size the student can find out how the system behaves over a reasonable number of periods, and what the corresponding averages are. This is a way to determine whether he understands the system.

B. The student can next attempt to build a model for the behavior of the system in the long run. He can use the computer program to test his model by comparing numerical results that he derives on paper with the numerical results derived on the computer.

C. Once the model of the system has been determined, the student can address himself to the problem of optimization: How to find the optimal values of the reorder point and the lot size. He must attempt to find decision rules leading to the desired values. Any numerical results that he obtains can be checked with the third type of output available from the computer program.

2. Presentation of Data

The student can control all the parameters in this program. Some will be fixed for several runs of the program, namely, the demand and the unit costs. These are entered in the primary input statement. The other parameters, which may vary with each run, are entered as secondary input. They include such variables as the reorder point, the lot size, the initial inventory, etc.

2.1 Demand and Unit Costs (Primary Input)

The demand, U, can take on any positive value. Its dimension is in quantity units per time period, where the quantity units may be a ton, gallon, etc., and where the time period may be a day, a week, a month, etc.

There are three types of unit costs. The unit carrying cost, C1, is in dollars per unit per period. The unit shortage cost, C2, is also in dollars per unit per period. The unit replenishing cost, C3, is in dollars per replenishment.

The request for primary input is in the form:

```
DEMAND,CARRY,SHORT,REPLEN
? < U,   C1,   C2,   C3 >
```

(The computer does not print the variable symbols, they are printed here as a visual aid to the student.)

Suppose the time period is one week and the quantity unit is a ton. Then, for example, we might have a demand of 5 tons per week, a unit carrying cost of $1 per ton per week, a unit shortage cost of

$9 per ton per week, and a unit replenishing cost of $36 per replenishment. Thus, the response to the Input statement would be:

```
DEMAND,CARRY,SHORT,REPLEN
?  5,  1,   9,   36
```

2.2 Decision Variables and Options (Secondary Input)

The program requires 10 secondary inputs during its running: the reorder point, S; the lot size, Q; the simulation index, X1; the initial inventory, Q1; the number of periods to be simulated, N, the number of periods to be printed, N1; the long term averages index, X2; **the total** costs index, X3; the reorder point and lot size increment, J1; and the primary input index, D.

Each of the indices X1, X2, X3, and D should be either 0 or 1.

If X1 = 1, then the simulation starts with an initial inventory of Q1 units and is carried out over N periods. The details of the first N1 ≤ N periods are printed. In addition, a summary of averages for the N periods is also printed. If N1 = 0, only the summary is printed.

When X1 = 0, the simulation option is not used. In that case, the values of Q1, N, and N1 are irelevant; however, they must be supplied.

If X2 = 1, then the long term averages summary is printed. This can be compared with the simulation summary. If X2 = 0, then the long term averages summary is not printed.

If X3 = 1, then a table of costs is printed in the neighborhood, J1, of the reorder point S and the lot size Q.

If D = 1, then at the end of the current run, the computer will ask for new primary input (i.e., new demand and unit costs will have to be supplied). Thereafter, or if D = 0, the computer will ask for new secondary input information. Thus, if the student wishes to retain the old primary input data and only change the secondary inputs on the next run of the program, he should enter 0 as the value for D.

The request for secondary input is in the form:

```
POINT,LOT,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
? < S, Q,  X1,  Q1, N,   N1,   X2,   X3,  J1,  D >
```

For example, the student may supply the following inputs:

```
POINT,LOT,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
?  -5, 20, 1,  10, 100,  8,     1,   1,   .5,  0
```

This input indicates that the student is interested in a reorder point of -5 and a lot size of 20. He wants a simulation with an initial inventory of 10 units to be run over 100 periods. He asks that the details for the first 8 periods (and a summary for the 100 periods) be printed. The student also asks for the long term averages summary. He also wants the table of total costs for reorder points of -5.5, -5, and -4.5 units and for lot sizes of 19.5, 20, and 20.5 units. Finally, he indicates that he would like another run with the same primary input.

3. Illustrations

The illustrations in this section are designed to exemplify all the capabilities of Program ISL-2. They also form the basis for the next section which deals with models and their solutions.

3.1 Balancing Carrying and Replenishing Costs

Examples 3.11, 3.12, and 3.13 illustrate the inputs and outputs relating to an inventory system in which the demand is 5 tons per month, the carrying cost is $1 per ton per month, and the replenishing cost is $36. No shortages are allowed. Hence, in the primary input the shortage cost is chosen as $99999 per month. In all examples the reorder point is 5 tons. This ensures that no shortages occur. The lot size in all examples is 20 tons. Therefore replenishments occur every 4 periods.

In Example 3.11 the secondary input calls for a simulation with an initial inventory of 15 tons. 10 periods are to be simulated and the details for all these periods are to be printed. The output is self-explanatory. Note that replenishments indeed occur whenever the reorder point is reached.

In Example 3.12 there is a request for a simulation over 100 periods and for the long term averages. The output is again self-explanatory. One observes that the simulation averages and

and long term averages are identical. They differ, though, from the results in Example 3.11.

The secondary input of Example 3.13 calls for total costs of reorder points of 0, 5, and 10 tons, and lot sizes of 15, 20, and 25 tons. The last entry in the input ensures a subsequent call for a new primary input. Note that the entry in the center of the table, $24 per period, is the same as the cost obtained in Example 3.12.

```
RUN


ISL-2      22:53    W2 SAT 12/23/67
     THE DETERMINISTIC REORDER POINT-LOT SIZE SYSTEM

DEMAND, CARRY, SHORT, REPLEN
?  5,      1,   99999,   36                                  Illustration 3.1


POINT,LOT SIZE, SIM, INIT, RUN, PRINTS, AVER, COSTS, STEP, DATA      Example 3.11
? 5,      20,       1,  15,  10,     10,     0,      0,    0,     0

REORDER POINT =   5                    LOT SIZE =   20

                             S I M U L A T I O N

PER.  BEGIN  DEMAND  END              CARRYING          SHORTAGES        REPLENISHMENTS
 1       15      5       10              12.5                0                   0
 2       10      5       5               7.5                 0                   1
 3       25      5       20              22.5                0                   0
 4       20      5       15              17.5                0                   0
 5       15      5       10              12.5                0                   0
 6       10      5       5               7.5                 0                   1
 7       25      5       20              22.5                0                   0
 8       20      5       15              17.5                0                   0
 9       15      5       10              12.5                0                   0
 10      10      5       5               7.5                 0                   1
```

```
     FOR A SIMULATION OF  10    PERIODS:
```

Example 3.11 (Cont'd)

```
                                    CARRYING      SHORTAGES   REPLENISHMENTS

AVERAGES                             14             0              .3
UNIT COSTS                           1              99999          36
COSTS PER PERIOD                     14             0              10.8

TOTAL COSTS PER PERIOD =  24.8


                **********************
```

```
POINT,LOT SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
? 5,    20,      1,  15, 100,  0,     1,     0,    0,    0
```

Example 3.12

```
REORDER POINT =  5               LOT SIZE =  20

                            S I M U L A T I O N


     FOR A SIMULATION OF  100   PERIODS:


                                    CARRYING      SHORTAGES   REPLENISHMENTS

AVERAGES                             15             0              .25
UNIT COSTS                           1              99999          36
COSTS PER PERIOD                     15             0              9

TOTAL COSTS PER PERIOD =  24


                  T H E   L O N G   T E R M   A V E R A G E S


                                    CARRYING      SHORTAGES   REPLENISHMENTS

AVERAGES                             15             0              .25
UNIT COSTS                           1              99999          36
COSTS PER PERIOD                     15             0              9

TOTAL COSTS PER PERIOD =  24
                **********************
```

```
POINT,LOT SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
? 5,    20,      0,   0,  0,   0,     0,     1,    5,    1
```

Example 3.13

```
REORDER POINT =  5               LOT SIZE =  20
```

Example 3.13 (Cont'd)

```
       T H E   T O T A L   C O S T   T A B L E

                            LOT SIZES
                 15            20            25
R.POINT
 0               19.5          19            19.7
 5               24.5          24            24.7
 10              29.5          29            29.7

                 . . . . . . . . . . . .     .
```

3.2 Balancing Shortage and Replenishing Costs

In Examples 3.21 and 3.22, as in the previous section, the demand is 5 tons per month, and the replenishing cost is $36. However, this time no inventory is to be carried and the shortage cost is $9 per ton per month. The appropriate entries have been made in the primary input.

The secondary inputs and the corresponding outputs in Examples 3.21 and 3.22 are self explanatory. Note the effect of the carrying cost on the total costs in the table of Example 3.22.

```
                 ++++++++++++++++++++++
DEMAND, CARRY, SHORT, REPLEN
?  5,  99999,  9,    36
```

Illustration 3.2

Example 3.21

```
POINT,LOT SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
? -10,  10,     1,   0, 10,   4,    0,    0,   0,   0

REORDER POINT = -10            LOT SIZE =  10

                   S I M U L A T I O N

PER.  BEGIN  DEMAND  END         CARRYING      SHORTAGES    REPLENISHMENTS
 1      0      5      -5            0             2.5             0
 2     -5      5     -10            0             7.5             1
 3      0      5      -5            0             2.5             0
 4     -5      5     -10            0             7.5             1
```

```
     FOR A SIMULATION OF  10   PERIODS:
```

Example 3.21 (Cont'd)

```
                                     CARRYING      SHORTAGES   REPLENISHMENTS

AVERAGES                              0              5              .5
UNIT COSTS                            99999          9              36
COSTS PER PERIOD                      0              45             18

TOTAL COSTS PER PERIOD =  63
```

```
                    ***********************

POINT,LOT SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
? -10,  10,     1,  0,  100,  0,    1,    1,   2.5,  1
```

Example 3.22

```
REORDER POINT = -10            LOT SIZE =  10

                            S I M U L A T I O N


     FOR A SIMULATION OF  100   PERIODS:


                                     CARRYING      SHORTAGES   REPLENISHMENTS

AVERAGES                              0              5              .5
UNIT COSTS                            99999          9              36
COSTS PER PERIOD                      0              45             18

TOTAL COSTS PER PERIOD =  63


                  T H E   L O N G   T E R M   A V E R A G E S
                                     CARRYING      SHORTAGES   REPLENISHMENTS

AVERAGES                              0              5              .5
UNIT COSTS                            99999          9              36
COSTS PER PERIOD                      0              45             18

TOTAL COSTS PER PERIOD =  63


          T H E   T O T A L   C O S T   T A B L E


                              LOT SIZES
                    7.5            10             12.5
R.POINT
-12.5               170.25         175.5          70.65
-10                 147.75         63             25050.2
-7.5                57.75          31293          100034.
```

3.3 Balancing Carrying and Shortage Costs

Examples 3.31 and 3.32 illustrate inventory systems in which only the reorder point is subject to control. This is the reason for selecting a zero cost of replenishing in the primary input, and for equating the demand and the lot size (5 tons).

The reader should attempt to explain the carrying and shortage averages. Otherwise, the outputs are self-explanatory.

Illustration 3.3

Example 3.31

```
                    ***********************
DEMAND,CARRY,SHORT,REPLEN
?  5,     1,    9,     0


POINT,LOT SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
? -2,      5,     1,  3,   10,  4,     0,     0,   0,    0

REORDER POINT = -2                  LOT SIZE =   5

                            S I M U L A T I O N

PER.  BEGIN  DEMAND  END               CARRYING          SHORTAGES      REPLENISHMENTS
 1      3      5     -2                  .9                .4                 1
 2      3      5     -2                  .9                .4                 1
 3      3      5     -2                  .9                .4                 1
 4      3      5     -2                  .9                .4                 1


     FOR A SIMULATION OF   10    PERIODS:


                                       CARRYING          SHORTAGES      REPLENISHMENTS

AVERAGES                                 .9                .4                 1
UNIT COSTS                               1                 9                  0
COSTS PER PERIOD                         .9                3.6                0

TOTAL COSTS PER PERIOD =   4.5


                    ***********************
                    ***********************


POINT,LOT SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
? -1,     5,      1,  4,   10,  0,     1,     1,    0.5,  1

REORDER POINT = -1             LOT SIZE =  5

                         S I M U L A T I O N


     FOR A SIMULATION OF  10    PERIODS:


                             CARRYING       SHORTAGES   REPLENISHMENTS

AVERAGES                      1.6             .1             1
UNIT COSTS                    1               9              0
COSTS PER PERIOD              1.6             .9             0

TOTAL COSTS PER PERIOD =  2.5


             T H E   L O N G   T E R M   A V E R A G E S


                             CARRYING       SHORTAGES   REPLENISHMENTS

AVERAGES                      1.6             .1             1
UNIT COSTS                    1               9              0
COSTS PER PERIOD              1.6             .9             0

TOTAL COSTS PER PERIOD =  2.5


       T H E   T O T A L   C O S T   T A B L E


                           LOT SIZES
              4.5          5              5.5
R.POINT
-1.5          3.25         3.25           3.29545
-1            2.36111      2.5            2.65909
-.5           2.02778      2.25           2.47727

                    ***********************
```

Example 3.32

3.4 Balancing Carrying, Shortage, and Replenishing Costs

In a certain sense, Examples 3.41 and 3.42 are extensions of all previous examples. No special constraints are imposed on the inventory system. The demand is 5 tons per month, the carrying cost is $1 per ton per month, the shortage cost is $9 per ton per month, and the replenishing cost is $36.

Previous examples seem to indicate that the optimal reorder point is about -1 ton and that the optimal lot size is 20 tons. However, the table of Example 3.42 shows that that optimum is elsewhere.

```
                    ***********************
DEMAND,CARRY,SHORT,REPLEN
? 5,     1,     9,     36
```

Illustration 3.4

Example 3.41

```
POINT,LOT SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
? -1,   20,      1,  14,  10,  10,    0,    0,   0,   0

REORDER POINT = -1               LOT SIZE =  20

                        S I M U L A T I O N

PER.  BEGIN  DEMAND  END          CARRYING        SHORTAGES     REPLENISHMENTS
 1      14     5      9             11.5             0                0
 2      9      5      4             6.5              0                0
 3      4      5     -1             1.6              .1               1
 4      19     5      14            16.5             0                0
 5      14     5      9             11.5             0                0
 6      9      5      4             6.5              0                0
 7      4      5     -1             1.6              .1               1
 8      19     5      14            16.5             0                0
 9      14     5      9             11.5             0                0
 10     9      5      4             6.5              0                0
```

Example 3.41 (Cont'd)

```
    FOR A SIMULATION OF  10    PERIODS:


                                  CARRYING      SHORTAGES   REPLENISHMENTS

AVERAGES                           9.02           .02           .2
UNIT COSTS                         1              9             36
COSTS PER PERIOD                   9.02           .18           7.2

TOTAL COSTS PER PERIOD =   16.4


                  *********************
```

Example 3.42

```
POINT,LOT SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
? -1,   20,     0,  0,   0,   0,     1,    1,    1,   1

REORDER POINT = -1              LOT SIZE =  20

             T H E   L O N G   T E R M   A V E R A G E S


                                  CARRYING      SHORTAGES   REPLENISHMENTS

AVERAGES                           9.025          .025          .25
UNIT COSTS                         1              9             36
COSTS PER PERIOD                   9.025          .225          9

TOTAL COSTS PER PERIOD =   18.25


         T H E   T O T A L   C O S T   T A B L E


                          LOT SIZES
                19            20             21
R.POINT
-2              18.0263       18.            18.0238
-1              18.2368       18.25          18.3095
 0              18.9737       19             19.0714

                  *********************
```

3.5 Special Features

In all illustrations considered thus far the inputs were such that the reorder point was always reached precisely. That is, we never had a case where the amount on hand at the end of a period was below the reorder point. Example 3.51 now illustrates such a case. Note also that the simulation results do not agree with the long term averages.

Example 3.51 also shows that the reorder point of -2 tons and a lot size of 20 tons provide a locally minimum cost of $18 per month.

In Example 3.52 we show how to change the basic period. In all previous examples the period was one month, or say, 30 days. In Example 3.52 the period is 6 days. Hence, a demand of 5 tons per month is equivalent to 1 ton per day. Similarly, we have a carrying cost of $0.2 per ton per 6 days, and a shortage cost of $1.8 per ton per 6 days. The replenishment cost of $36 is not changed.

Note now that the simulation results are identical with the long term averages. Also note that all costs are one-fifth of the costs in Example 3.51, since the period is now 6 days instead of 30 days.

```
                                  ************************
DEMAND, CARRY, SHORT, REPLEN
?  5,      1,      9,      36
```

Example 3.51

Example 3.51(Cont'd)

```
OINT,LOT SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
: -2,    20,        1,  14,  100, 10,    1,        1,   0.1,  1

REORDER POINT = -2              LOT SIZE =   20

                    S I M U L A T I O N

PER.  BEGIN  DEMAND  END          CARRYING       SHORTAGES     REPLENISHMENTS
 1      14     5      9            11.5            0                0
 2      9      5      4            6.5             0                0
 3      4      5     -1            1.6             .1               0
 4     -1      5     -6            0               3.5              1
 5      14     5      9            11.5            0                0
 6      9      5      4            6.5             0                0
 7      4      5     -1            1.6             .1               0
 8     -1      5     -6            0               3.5              1
 9      14     5      9            11.5            0                0
 10     9      5      4            6.5             0                0


     FOR A SIMULATION OF  100   PERIODS:


                                  CARRYING       SHORTAGES     REPLENISHMENTS

AVERAGES                          4.9            .9               .25
UNIT COSTS                        1              9                36
COSTS PER PERIOD                  4.9            8.1              9

TOTAL COSTS PER PERIOD =   22.


              T H E   L O N G   T E R M   A V E R A G E S

                                  CARRYING       SHORTAGES     REPLENISHMENTS

AVERAGES                          8.1            .1               .25
UNIT COSTS                        1              9                36
COSTS PER PERIOD                  8.1            .9               9

TOTAL COSTS PER PERIOD =   18.


         T H E   T O T A L   C O S T   T A B L E


                              LOT SIZES
                 19.9          20              20.1
R.POINT
-2.1             18.0033       18.0025         18.0022
-2               18.0003       18.             18.0002
-1.9             18.0023       18.0025         18.0032

                 ***********************
```

```
DEMAND, CARRY, SHORT, REPLEN
?  1,    0.2,   1.8,    36
```

Example 3.52

```
POINT,LOT  SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
? -2,    20,       1,  4,  100,  8,      1,     1,  0.1,  1


REORDER POINT = -2              LOT SIZE =  20


                          S I M U L A T I O N


PER.  BEGIN  DEMAND  END          CARRYING      SHORTAGES    REPLENISHMENTS
 1      4      1      3            3.5            0              0
 2      3      1      2            2.5            0              0
 3      2      1      1            1.5            0              0
 4      1      1      0            .5             0              0
 5      0      1     -1            0             .5              0
 6     -1      1     -2            0             1.5             1
 7     18      1      17           17.5           0              0
 8     17      1      16           16.5           0              0


     FOR A SIMULATION OF  100   PERIODS:


                                  CARRYING      SHORTAGES    REPLENISHMENTS

AVERAGES                           8.1            .1             .05
UNIT COSTS                         .2             1.8            36
COSTS PER PERIOD                   1.62           .18            1.8

TOTAL  COSTS PER PERIOD =   3.6


                  T H E   L O N G   T E R M   A V E R A G E S
                                  CARRYING      SHORTAGES    REPLENISHMENTS

AVERAGES                           8.1            .1             .05
UNIT COSTS                         .2             1.8            36
COSTS PER PERIOD                   1.62           .18            1.8

TOTAL  COSTS PER PERIOD =   3.6


            T H E   T O T A L   C O S T   T A B L E


                              LOT SIZES
                   19.9          20            20.1
R.POINT
-2.1               3.60065       3.6005        3.60045
-2                 3.60005       3.6           3.60005
-1.9               3.60045       3.6005        3.60065
```

The change of period length is also illustrated in Examples 3.53 and 3.54. The examples deal with an inventory system in which the demand is 2400 parts per year, the carrying cost is $0.56 per part per year, and the replenishing cost is $42. No shortages are allowed.

One can show that the optimal reorder point is 0 and that the optimal lot size is 600 parts. Note the meaningless output in the simulation of Example 3.53 and how this is corrected in Example 3.54.

<u>Example 3.53</u>

```
                     . . . . . .******************
DEMAND, CARRY, SHORT, REPLEN
? 2400, 0.56,99999,  42


POINT,LOT SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
?  0,   600,    1,  600, 10,  5,    1,    1,   10,  1

REORDER POINT =  0                LOT SIZE =  600

                       S I M U L A T I O N

PER. BEGIN DEMAND END          CARRYING       SHORTAGES    REPLENISHMENTS
 1     600   2400     -1800     75              675               1
 2    -1200      2400     -3600                  0              2400
 1
 3    -3000      2400     -5400                  0              4200
 1
 4    -4800      2400     -7200                  0              6000
 1
 5    -6600      2400     -9000                  0              7800
 1
```

:

Example 3.53 (Cont'd)

```
FOR A SIMULATION OF  10    PERIODS:


                           CARRYING      SHORTAGES   REPLENISHMENTS

AVERAGES                   7.5           8707.5          1
UNIT COSTS                 .56           99999           42
COSTS PER PERIOD           4.2           870741292       42

TOTAL COSTS PER PERIOD =  870741338


             T H E   L O N G   T E R M   A V E R A G E S


                           CARRYING      SHORTAGES   REPLENISHMENTS

AVERAGES                   300.          0               4
UNIT COSTS                 .56           99999           42
COSTS PER PERIOD           168.          0               168

TOTAL COSTS PER PERIOD =  336.


         T H E   T O T A L   C O S T   T A B L E


                        LOT SIZES
                590         600           610
R.POINT
-10            8804.99     8663.7        8527.13
 0             336.047     336.          336.046
 10            341.647     341.6         341.646

               ***********************
DEMAND,CARRY,SHORT,REPLEN
? 200,0.046667,99999, 42
```

Example 3.54

```
POINT,LOT SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA
? 0,   600,    1,  400, 12,  6,    1,    1,   10,  1


REORDER POINT =  0              LOT SIZE =  600
```

Example 3.54 (Cont'd)

```
                         S I M U L A T I O N

PER.  BEGIN  DEMAND  END          CARRYING       SHORTAGES      REPLENISHMENTS
 1     400    200    200           300              0                0
 2     200    200    0             100              0                1
 3     600    200    400           500              0                0
 4     400    200    200           300              0                0
 5     200    200    0             100              0                1
 6     600    200    400           500              0                0


     FOR A SIMULATION OF   12     PERIODS:


                                  CARRYING       SHORTAGES      REPLENISHMENTS

AVERAGES                           300              0                .333333
UNIT COSTS                         .046667          99999            42
COSTS PER PERIOD                   14.0001          0                14.

TOTAL COSTS PER PERIOD =   28.0001


               T H E   L O N G   T E R M   A V E R A G E S


                                  CARRYING       SHORTAGES      REPLENISHMENTS

AVERAGES                           300              0                .333333
UNIT COSTS                         .046667          99999            42
COSTS PER PERIOD                   14.0001          0                14.

TOTAL COSTS PER PERIOD =   28.0001


          T H E   T O T A L   C O S T   T A B L E


                             LOT SIZES
                  590           600            610
R.POINT
-10               8502.03       8360.79        8224.18
 0                28.0041       28.0001        28.0039
 10               28.4707       28.4668        28.4706

                    ***********************
DEMAND,CARRY,SHORT,REPLEN
Z STOP

RAN      30 SEC.
```

4. Construction and Solution of the Model

A thorough understanding of how the system behaves is necessary before the student can construct a model of the system. This understanding is achieved by the use of the simulation option of the program.

Once the system is completely understood, one should attempt to predict the long term averages for any primary input and any reorder points and lot sizes. This is the essence of constructing a mathematical model of the system. The student can check his model by comparing numerical results of his model with those given by the program.

After the model has been constructed, the student must find an analytical method for the determination of the optimal reorder point and lot size. Although he may use the costs option of the program to test his optimizing technique, the student must mathematically justify his results - specific precaution should be taken to eliminate the possibility of assuming that a local minimum is the global minimum.

4.1 Understanding the System

A good way to understand a system is to see it work. The student should therefore select some values for U, S, and Q and graphically describe the progress of the system as a function of time. How much inventory is carried (or is in shortage)? when is there a replenishment? etc. His selection of values should reflect the following situations:

(a) No shortages are allowed ($S \geq 0$).

(b) No inventory is carried ($S + Q \leq 0$).

(c) Inventory is carried and shortages are allowed ($S < 0$ and $S + Q > 0$).

Next, he should calculate the three averages: inventory carried, shortages, and replenishments. These should be checked against the results that the computer gives in the simulation averages.

4.2 Construction of the Model

The general model of the system may be represented by the equation

$$C(s,q) = c_1\, I_1(s,q,r) + c_2\, I_2(s,q,r) + c_3\, I_3(s,q,r) \qquad (1)$$

where

s = reorder point

q = lot size

C = total cost per unit time

c_1= carrying cost per unit per unit time

c_2= shortage cost per unit per unit time

c_3= replenishing cost

I_1= average amount carried

I_2= average shortage

I_3= average number of replenishments per unit time.

r = demand per unit time

To find I_1, I_2, I_3, for any s, q, and r, one may wish to consider the following cases:

(a) No Shortages are Allowed

This case is equivalent to stating that c_2 is infinitely large. It should be obvious that $s \geq 0$, hence the model is:

$$C(s,q) = c_1 I_1(s,q,r) + c_3 I_3(s,q,r) \qquad s \geq 0 \qquad (2)$$

The student should now show that I_1 is a linear function of s and q, and that I_3 is not a function of s.

(b) No Inventory is Carried

This case is equivalent to stating that c_1 is infinitely large. The model is now:

$$C(s,q) = c_2 I_2(s,q,r) + c_3 I_3(s,q,r) \qquad s + q \leq 0 \qquad (3)$$

I_2 should be shown to be a linear function of s and q, and I_3 to be as in Case (a).

(c) Prescribed Lot Size

In this case the lot size is fixed, say, at $q_p = rt$, when t is the length of the period. Then I_3 is fixed too. Assume then that $c_3 = 0$. The model becomes

$$C(s,q_p) = c_1 I_1(s,q_p) + c_2 I_2(s,q_p) \qquad (4)$$

I_1 and I_2 should be determined for three distinct situations: (1) $s \geq 0$, (2) $s + q_p \leq 0$, and (3) $s \leq 0$ and $s + q_p \geq 0$. The results for the first two situations correspond to Cases (a) and (b) respectively. In situation (3) one has to show that I_1 and I_2 are quadratic functions of s.

(d) The General Case

After constructing the models of Cases (a), (b), and (c), the general model of Equation (1) can be immediately constructed. The model should be stated in the form:

$$C(s,q) = \begin{cases} C^a & s \geq 0 \\ C^c & s \leq 0 \quad s + q \geq 0 \\ C^b & s + q \leq 0 \end{cases} \qquad (5)$$

where C^a and C^b are as in Equations (2) and (3) respectively and C^c is in part as in Equation (4).

As the models are being constructed the student can check them with the long term averages option of the program. The examples in the first four illustrations of the previous section correspond respectively to Cases (a) to (d).

4.3 Solution of the Model

After the model of a system has been constructed one generally wishes to find the optimal values of the controllable variables. The process of finding these values is referred to as the solution of the model. Thus we are interested in finding the optimal values of the reorder point and the lot size. We will refer to these values as s_o and q_o.

It is again advisable to consider the four cases described in the previous section.

In Case (a) the student should show that $s_o = 0$ and that q_o is a function of r, c_1, and c_3.

In Case (c), the student should show that s_o is a function of q_p, c_1, and c_2.

Finally, in Case (d), he should show that s_o is not positive and a function of r, c_1, c_2, and c_3, and that q_o is another function of r, c_1, c_2, and c_3.

The optimal functions which the student derives may be checked using the table of total costs of the program. See the details in Examples 3.13, 3.22, 3.32, 3.42, 3.51, 3.52, 3.53, and 3.54.

5. Extensions and Problems for Solution

5.1 Extensions

By changing some of the properties of the inventory system of Program ISL-2, a variety of new systems can be obtained. For example, if demand is allowed to vary and follows some given probability distribution, we would have the probabilistic reorder point - lot size system. This has been done in Program ISL-3 which is described in another manual. Less extensive extensions are stated in the form of exercises.

A. Program ISL-2 assumes that shortages are made up - this is the classical back-order case. What changes should be made in the program for the lost-sales case, in which shortages are not made up.

B. Extend Program ISL-2 for systems in which leadtime is L periods. The leadtime should be specified with the primary output.

C. Change Program ISL-2 so that the controllable variables are the scheduling period t and an order level S (instead of the reorder point s and the lot size q). In the new system replenishments occur every t periods. The replenishment raises inventory to a level S.

D. In Program ISL-2 the lot size q is added to inventory instantaneously. Extend the program to allow the addition to stock to be at a rate p.

E. Extend Program ISL-2 to an inventory system with 2 items. For the first item the rate of demand is r_1, the carrying cost is c_{11} per unit per unit time, and the shortage cost is c_{21} per unit per unit time.

The corresponding parameters for the second item are r_2, c_{12}, and c_{22}. The replenishment cost is c_3. This cost is incurred whenever one or both items are replenished.

5.2 Problems for Solution

A. In Example 3.51 there is a marked difference between the simulation results and the long term averages.

   (1) Explain the reason for the difference.

   (2) What would be the simulation results for initial inventories of 15? 16? 17? 18? 19?

B. Explain the simulation results in Example 3.53.

C. Find the optimal solution of a deterministic reorder point lot size system in which the demand is 6 lb. per month, the carrying cost is $0.75 per pound per month, the shortage cost is $1.50 per pound per month, and the replenishing cost is $54.00.

D. Solve Problem C above if the respective quantities are 25 lb., $9 per pound per month, $16 per pound per month, and $288.

E. Solve Problem D when the reorder point and the lot size must be integer multiples of 20 lb.

```
ISL-2

100 REM BY E.NADDOR AND RICHARD SACHER, THE JOHNS HOPKINS UNIVERSITY
110
140PRINT"    THE DETERMINISTIC REORDER POINT-LOT SIZE SYSTEM"
150PRINT
160PRINT
164
165 REM PRIMARY INPUT
170 PRINT"DEMAND,CARRY,SHORT,REPLEN"
180 INPUT U,C1,C2,C3
220  GOSUB 7000
230 GOTO 1050
999
1000 REM SECONDARY INPUT
1010 GOSUB 7000
1040
1050 PRINT"POINT,LOT SIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS,STEP,DATA"
1060INPUT S,Q,X1,Q1,N,N1,X2,X3,J1,D
1070PRINT
1080PRINT
1090PRINT"REORDER POINT = "S,"LOT SIZE = "Q
1130PRINT
1140PRINT
1150 IF X1=1 THEN 2000
1160 IF X2=1 THEN 3000
1170 IF X3=1 THEN 4000
1171 PRINT"                 ***********************"
1175 IF D=1 THEN 165
1180 GOTO 1000
1999
2000PRINT"                        S I M U L A T I O N"
2002 LET A1=0
2004 LET A2=0
2006 LETA3=0
2010 IF N1=0 THEN 2050
2020PRINT
2030PRINT
2040PRINT"PER. BEGIN DEMAND END        CARRYING     SHORTAGES   REPLENI
2050 FOR I=1 TO N
2051 IF Q1<=0 THEN 2059
2052 IF (Q1-U)<0 THEN 2056
2053 LET I1=Q1-U/2
2054 LET I2=0
2055 GOTO 2061
2056 LET I1=Q1↑2/(2*U)
2057 LET I2=(Q1-U)↑2/(2*U)
2058 GOTO 2061
2059 LET I1=0
2060 LET I2=-Q1+U/2
2061 LET Q2=Q1-U
2062 IF Q2>S THEN 2066
```

ISL-2 CONTINUED

```
2063 LET I3=1
2064 LET Y=Q2+Q
2065 GO TO 2069
2066 LET I3=0
2067 LET Y=Q2
2069 IF I>N1 THEN 2080
2070PRINT I;Q1;U;Q2,I1,I2,I3
2080 LET Q1=Y
2082 LET A1=A1+I1
2084 LET A2=A2+I2
2086 LET A3=A3+I3
2090 NEXT I
2100 LET I1=A1/N
2110 LET I2=A2/N
2120 LET I3=A3/N
2121 PRINT
2122 PRINT
2123 PRINT
2125 PRINT "     FOR A SIMULATION OF "N; " PERIODS:"
2130 GOSUB 6000
2320 GOSUB 7000
2330 GOTO 1160
2999
3000PRINT"              T H E   L O N G   T E R M   A V E R A G E S"
3002 PRINT
3003 GOSUB 5000
3005 GOSUB 6000
3100 GOTO 1170
3999
4000 REM COST TABLE
4002 LET S9=S
4004 LET Q9=Q
4006 LET S(1)=S9-J1
4010 LET S(2)=S9
4020 LET S(3)=S9+J1
4030 LETQ(1)=Q9-J1
4040 LET Q(2)=Q9
4050 LET Q(3)=Q9+J1
4060 GOSUB 7000
4070 PRINT"       T H E   T O T A L   C O S T   T A B L E"
4080PRINT
4090PRINT
4140PRINT"                             LOT SIZES"
4150PRINT"  ",Q(1),Q(2),Q(3)
4160PRINT"R.POINT"
4170 FOR I=1 TO 3
4180 LET S=S(I)
4190 PRINT S,
4200 FOR J=1 TO 3
4210 LET Q=Q(J)
```

ISL-2 CONTINUED

```
4220 GOSUB 5000
4230 PRINT C1*I1+C2*I2+C3*I3,
4240 NEXT J
4250 PRINT
4260 NEXT I
4261 PRINT
4270 GO TO 1171
4999
5000 REM I1,I2,I3
5005 LET I3=U/Q
5010 IF S<0 THEN 5050
5020 LET I1=S+Q/2
5030 LET I2=0
5040 RETURN
5050 IF S<-Q THEN 5090
5060 LET I1=(Q+S)↑2/(2*Q)
5070 LET I2=S↑2/(2*Q)
5080 RETURN
5090 LET I1=0
5100 LET I2=-S+Q/2
5110 RETURN
5999
6000 REM AVERAGES AND COSTS
6005 GOSUB 7000
6010PRINT"                                    CARRYING      SHORTAGES    REPLENI
6020 PRINT
6040PRINT"AVERAGES","  ",I1,I2,I3
6050PRINT"UNIT COSTS","  ",C1,C2,C3
6060PRINT"COSTS PER PERIOD",C1*I1,C2*I2,C3*I3
6070PRINT
6080PRINT"TOTAL COSTS PER PERIOD = "C1*I1+C2*I2+C3*I3
6170 RETURN
6999
7000REM 3 SPACE SUB
7010 PRINT
7020 PRINT
7030 PRINT
7040 RETURN
7999
9999 END
```

INVENTORY SYSTEMS LABORATORY THREE (ISL-3)

Manual 3A

by

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October 1967

Supported by the Office of Education

Contract OEC2-7-070015-3111

1. Introduction

This laboratory deals with System 3 which is an extension of System 2. Whereas in System 2 demands are known and constant, in System 3 demands are probabilistic. Otherwise the two systems are identical. As in ISL-2, the program can be used as follows:

A. For any reorder point and lot size the student can find out how the system behaves over a reasonable number of periods, and what the corresponding averages are. This is a way to determine whether he understands the system.

B. The student can next attempt to build a model for the expected behavior of the system in the long run. The The construction of the model will probably prove to be a challenge. The student can use the computer program to test his model by comparing numerical results that he derives on paper with the numerical results derived on the computer.

C. Once the model of the system has been determined, the student addresses himself to the problem of optimization: How to find the optimal values of the reorder point and the lot size. This is strictly a mathematical problem for which it is rather difficult to get a closed form solution. The student must attempt to find an algorithm leading to the desired values. Any numerical results that he obtains can be checked with the third type of output available from the the computer program.

2. The Probabilistic Reorder Point - Lot Size System

2.1 Demands

The demand X during a reviewing period is a random variable which can take discrete values; 0, U, 2U, etc., up to the maximum demand of X2. The probability of demand, P(X), must be given. The student supplies the value of U, X2, P(0), P(U),...,P(X2).

2.2 Unit Costs

The cost of carrying inventory is C1 per unit per reviewing period. The cost of shortage is C2 per unit per reviewing period. The cost of replenishing is C3 per replenishment. The student supplies the values of C1, C2, and C3.

2.3 Output

The program allows for three types of outputs:

A. Simulation. For a run of size M the inventory system is simulated by giving M demands from the probability distribution supplied by the student. The simulation provides detailed printouts for the first T periods. For each of these periods the average amount carried, the average shortage, and an indication of whether there was a replenishment are supplied. At the end of the simulation the overall averages and the corresponding costs are printed. The student has to supply the values of the initial inventory IO, the run size M, and the number of desired printouts T.

B. Long Term Averages. If this option is indicated the computer prints the expected values of the average inventory, average shortage, average number of replenishments per unit time, and the corresponding expected costs.

C. A Table of Expected Total Costs. For this option expected costs in the neighborhood of the specified reorder point and lot size are printed.

2.4 Decisions

The student can control the reorder point S1 and the lot size Q. These variables, as well as the variables controlling the output, are supplied by the student as input during the running of the program. When the third output option is elected the expected total costs are given for the values of S1-U, S1, and S1+U, and Q-U, Q, and Q+U.

2.5 Objectives

The student supplies the unit costs and the probability distribution of demand in data statements. Thereafter, he has three major objectives:

A. To check whether he understands how the system behaves using the simulation option of the program.

B. To construct the model of the system and to use the model to determine numerical values of the expected average inventory which will be carried, the expected shortages, the expected number of replenishments, and the expected total cost of the system. The long term averages option of the program allows him to check whether his model indeed describes the system.

C. To develop an algorithm for finding the optimal values of the reorder point and the lot size. The table of expected total costs can be used to check the algorithm.

3. Presentation of Data and Inputs

3.1 Unit Costs: C1, C2, C3

Three unit costs have to be supplied by the student. The unit carrying cost is in dollars per unit per reviewing period. The unit shortage cost is also in dollars per unit per reviewing period. The unit replenishing cost is in dollars. The general data statement is:

```
9010 DATA < C1, C2, C3 >
```

For example, if the unit carrying cost is $5 per ton per week, the unit shortage cost is $50 per ton per week, and the unit replenishing cost is $40, the data statement reads:

```
9010 DATA 5, 50, 40
```

3.2 Probability Distribution of Demand: P(0), P(U),...,P(X2)

Demand can take the values of 0,U,2U,...,X2-U,X2. The corresponding probabilities are P(0),...,P(X2). The general data statement is in form:

```
9020 DATA < U,X2,P(0),P(U),...,P(X2) >
```

For example, for demands of 0,2,4,6, and 8, with corresponding probabilities of .05,.24,.38,.21, and .12, the data statement is:

```
9020 DATA 2,8,.05,.24,.38,.21,.12
```

Obviously, the sum of the probabilities must be equal to 1. The cumulative probability distribution is computed by the program and is printed immediately. If the cumulative probability distribution of X2 is not equal to 1, the student should stop the execution of the program and check his data statements.

3.3 Inputs

The program requires 8 inputs during its running: the reorder point S1, the lot size Q, an index Y related to simulation, the initial inventory IO, the simulation run size M, the number of periods to be printed T, an index S related to long term averages, and an index R related to the table of expected costs.

Each of the indices Y,S, or R should be either 0 or 1.

When Y=0, the simulation option is not used. In that case, the values of IO, M, and I are irrelevant; however, they must be supplied.

If Y=1, then the simulation starts with an initial inventory of IO units, it is carried out over M periods, and the details of the first T periods are printed. If T=0, only the averages for the simulation are printed.

If S=1, then the long term averages are printed. (These can be compared with the simulation averages.) If S=0, then the long term averages are not printed.

If R=1, then a table of expected costs is printed in the neighborhood of the reorder point S1 and the lot size Q. Otherwise such a table is not printed.

The request of inputs is in the form:

```
  POINT, LOT SIZE, SIM, INIT, RUN, PRINTS, AVER, COSTS
? <  S1 ,    Q    , Y , IO , M ,   T   , S ,   R  >
```

For example, the following inputs may be supplied:

```
POINT, LOT SIZE, SIM, INIT, RUN, PRINTS, AVER, COSTS
? 0 ,   10   , 1 , 10 , 100,  5   , 1 ,  1
```

In this example the student is interested in a reorder point of 0, and a lot size of 10. He wants a simulation with an initial investing of 10 to be run over 100 periods. He asks that the details for the first 5 periods be printed. He also wants the long term averages and a table of expected costs.

4. Illustrations

Two illustrations are given in this section. The first illustrates the full capabilities of Program ISL-3. The second deals with a deterministic reorder point - lot size system and allows a comparison with Program ISL-2.

4.1 Illustration 1

The reviewing period in a reorder point - lot size system is one week. The carrying cost is \$5 per pound per week, the shortage cost is \$50 per pound per week, and the ordering cost is \$40 per order.

Demand may be assumed to occur in units of 2 pounds, with a maximum of 8 pounds in any week. The demand probabilities may be assumed to be P(0) = .05, P(2) = .24, P(4) = .38, P(6) = .21 and P(8) = .12.

For some reorder points and lot sizes one wishes to study the behavior of the system and to check long term averages. One also wants to check whether some specified reorder point and lot size give a minimum total expected cost.

The data, the computer outputs, the inputs, and the results are given on Pages 8 to 10.

```
9010 DATA 5,50,40
9020 DATA 2,8, .05, .24, .38, .21, .12
RUN


ISL-3*    21:36   W2 WED 10/25/67

  THE PROBABILISTIC REORDER POINT- LOT SIZE SYSTEM

CARRYING COST = 5    PER UNIT PER PERIOD
SHORTAGE COST = 50   PER UNIT PER PERIOD
REPLENISHING COST = 40   PER REPLENISHMENT

DEMAND            PROBABILITY    CUMULATIVE
 0                 .05            .05
 2                 .24            .29
 4                 .38            .67
 6                 .21            .88
 8                 .12            1.

************************************************************

POINT,LOTSIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS
?  -4,  14,    1,  10,  10,  10,     0,    0
```

Example 4.11

```
REORDER POINT =-4    LOT SIZE = 14



                     SIMULATION

PER   BEGIN DEMAND END          CARRYING        SHORTAGES   REPLENISHMENTS

 1     10     6     4            7               0             0
 2     4      2     2            3               0             0
 3     2      2     0            1               0             0
 4     0      4    -4            0               2             1
 5     10     2     8            9               0             0
 6     8      4     4            6               0             0
 7     4      2     2            3               0             0
 8     2      8    -6            .25             2.25          1
 9     8      4     4            6               0             0
 10    4      2     2            3               0             0
FOR A SIMULATION OF 10    PERIODS:


                DEMAND          CARRYING        SHORTAGES    REPLENISHMENTS

AVERAGES         3.6            3.825           .425           .2
UNIT COSTS                      5               50             40
COSTS PER PERIOD                19.125          21.25          8.

TOTAL COST PER PERIOD = 43.375

************************************************************
```

Example 4.12

```
POINT,LOTSIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS
?  -4,    14,    1,  10, 500,   10,    1,     1

REORDER POINT =-4     LOT SIZE = 14


                    SIMULATION

PER    BEGIN DEMAND END           CARRYING          SHORTAGES    REPLENISHMENTS

 1      10     2     8            9                 0                0
 2      8      6     2            5                 0                0
 3      2      4    -2            .5                .5               0
 4     -2      6    -8            0                 5                1
 5      6      4     2            4                 0                0
 6      2      8    -6            .25               2.25             1
 7      8      8     0            4                 0                0
 8      0      6    -6            0                 3                1
 9      8      4     4            6                 0                0
 10     4      4     0            2                 0                0

FOR A SIMULATION OF 500   PERIODS:

                 DEMAND           CARRYING          SHORTAGES    REPLENISHMENTS

AVERAGES          4.172           2.88067           1.00267          .298
UNIT COSTS                        5                 50               40
COSTS PER PERIOD                  14.4033           50.1333          11.92

TOTAL COST PER PERIOD = 76.4567

                LONG TERM AVERAGES

                 DEMAND           CARRYING          SHORTAGES    REPLENISHMENTS

AVERAGES          4.22            2.91571           1.02571          .301429
UNIT COSTS                        5                 50               40
COSTS PER PERIOD                  14.5786           51.2857          12.0571

TOTAL COST PER PERIOD = 77.9214


              THE EXPECTED TOTAL COST TABLE

   LOT SIZE =      12              14               16
R.POINT
-6                 130.342         115.929          106.369
-4                 84.3333         77.9214          74.3625
-2                 56.6583         55.6286          56.1062

*******************************************************************
```

Example 4.13

```
POINT,LOTSIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS
?  2,    8,     0,    0,   0,    0,     0,    1


REORDER POINT = 2    LOT SIZE = 8



          THE EXPECTED TOTAL COST TABLE

   LOT SIZE =    6                 8                  10
R.POINT
 0              53.5833           48.75              46.89
 2              50.0167           48.575             48.75
 4              56.5333           55.9625            56.66

*******************************************************************
```

Example 4.14

```
POINT,LOTSIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS
?  0,    10,     1,   4,   1000,   0,    1,    1


REORDER POINT = 0    LOT SIZE = 10



                     SIMULATION


FOR A SIMULATION OF 1000      PERIODS:


                   DEMAND          CARRYING          SHORTAGES     REPLENISHMENTS

AVERAGES            4.336           3.97667           .214667          .434
UNIT COSTS                          5                 50               40
COSTS PER PERIOD                    19.8333           10.7333          17.36

TOTAL  COST PER PERIOD = 47.9767


                LONG TERM AVERAGES


                   DEMAND          CARRYING          SHORTAGES     REPLENISHMENTS

AVERAGES           4.22             4.082             .192             .422
UNIT COSTS                          5                 50               40
COSTS PER PERIOD                    20.41             9.6              16.88

TOTAL COST PER PERIOD = 46.89


          THE EXPECTED TOTAL COST TABLE

   LOT SIZE =    8                 10                 12
R.POINT
-2              67.7625           60.1               56.6583
 0              48.75             46.89              47.3167
 2              48.575            48.75              50.5333
```

Examples 4.11, 4.12, 4.13, and 4.14 illustrate the following points:

a. The simulated demands follow the given distribution. Different simulations use different sequences of demands (see Examples 4.11 and 4.12). The mean demand is 4.22 (see Example 4.12 under Long Term Averages). However, the simulated mean demand varies even for large simulations (in Example 4.12, for a simulation of 500, it is 4.172, while in Example 4.14, for a simulation of 1000, it is 4.336).

b. The method of obtaining simulation averages and costs is illustrated in Example 4.11, where only 10 periods are simulated. This example also clearly illustrates the behavior of the system from period to period, the resulting average amounts that are carried and/or are short, and whether replenishments are ordered (and delivered).

c. The method for computing long term averages is not illustrated, of course. This is one of the principal tasks the student has to work on.

d. Examples 4.12 and 4.14 can be used to compare simulation averages and long term averages.

e. Example 4.13 provides an illustration of a reorder point S1 and a lot size Q which are, in a certain sense, locally optimal, since

$$C(S1,Q) \le C(S1 \pm U,Q) \quad \text{and} \quad C(S1,Q) \le C(S1,Q \pm U)$$

As can be seen from Example 4.14, the optimal solution appears to be S1=0 and Q = 10, with a minimum cost of $46.89 per week.

4.2 Illustration 2

Program ISL-3 can handle both probabilistic and deterministic reorder point - lot size systems. However, as compared with program ISL-2, it is not flexible for deterministic systems in the display of The Expected Total Cost Table. In Program ISL-2 the step for various reorder points and lot sizes can be specified with the input. In Program ISL-3 the step equals the basic demand unit U.

Examples 4.21 and 4.22 illustrate a system with deterministic demand of, say, 5 tons per month. The expected costs in Example 4.21 should then be interpreted to be in dollars per month. If one desired a smaller basic quantity unit, say, 1 ton, then the basic time unit would have to be 30/5 = 6 days. For this time unit the data and results are as in Example 4.22. The costs are now in dollars per 6 days. Thus the cost for a reorder point of -2 and a lot size of 20 is \$3.6 x 5 = \$18 per month.

5. Construction and Solution of the Model

The properties of the inventory system to be studied can be summarized as follows: Demand x during some reviewing period w is uniform and follows a discrete probability distribution P(0), P(u), P(2u),...,P(x_{max}) where u is some basic unit and x_{max} is the maximum demand during w. The carrying cost is c_1 per unit per unit time, the shortage cost is c_2 per unit per unit time, and the replenishing cost is c_3.

Example 4.21

```
9010 DATA 1,9,36
9020 DATA 5,5,0,1
RUN


ISL-3*    21:55   W2 WED 10/25/67

  THE PROBABILISTIC REORDER POINT- LOT SIZE SYSTEM

CARRYING COST = 1     PER UNIT PER PERIOD
SHORTAGE COST = 9     PER UNIT PER PERIOD
REPLENISHING COST = 36   PER REPLENISHMENT

DEMAND              PROBABILITY    CUMULATIVE
 0                  0              0
 5                  1              1

**************************************************************

POINT,LOTSIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS
? 0,   20,    0,   0,  0,    0,      0,   1


REORDER POINT = 0     LOT SIZE = 20




            THE EXPECTED TOTAL COST TABLE

   LOT SIZE =    15                20                25
R.POINT
-5               22.8333           20.25             19.7
 0               19.5              19                19.7
 5               24.5              24                24.7

**************************************************************

POINT,LOTSIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS
? STOP

RAN      6 SEC.
```

Example 4.22

```
9310 DATA .2, 1.8, 36
9320 DATA 1,1,0,1
RUN


ISL-3*    22:00   #2 WED 10/25/67

  THE PROBABILISTIC REORDER POINT--LOT SIZE SYSTEM

CARRYING COST = .2        PER UNIT PER PERIOD
SHORTAGE COST = 1.8       PER UNIT PER PERIOD
REPLENISHING COST = 36     PER REPLENISHMENT

DEMAND             PROBABILITY    CUMULATIVE
 0                  0              0
 1                  1              1

***********************************************************

POINT,LOTSIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS
? -2,   20,    0,  0,  0,  0,    0,  1


REORDER POINT =-2    LOT SIZE = 20



           THE EXPECTED TOTAL COST TABLE

   LOT SIZE =    19              20              21
R.POINT
-3               3.66842         3.65            3.64286
-2               3.60526         3.6             3.60476
-1               3.64737         3.65            3.6619

***********************************************************

POINT,LOTSIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS
? STOP

RAN       5 SEC.
```

An(s,q) policy is used. That is, inventories are reviewed every period w. When the amount on hand is s or below, a replenishment of q units is added to inventory immediately. If the total amount is still at s or below, another q units are added. As many replenishments of q are added until the amount on hand is larger than s. However, only one replenishment cost of c_3 is incurred, no matter how many q's are needed to raise inventory above s.

The model of the system can be represented by:

$$C(s,q) = c_1 I_1(s,q) + c_2 I_2(s,q) + c_3 I_3(s,q)$$

where C is the expected total cost of the system per unit of time, I_1 and I_2 are respectively the average amount carried and the average shortage, and I_3 is the number of replenishments per unit time.

The solution of the model is a pair of optimal values s_o, q_o which minimize C. Namely

$$C(s_o,q_o) \le C(s,q)$$

5.1 Construction of the Model

To find C(s,q) one only needs to find $I_1(s,q)$, $I_2(s,q)$, and $I_3(s,q)$. It is suggested that the student first find $I_3(s,q)$. He should then find $I_1(s,q)$ when $I_2(s,q) = 0$. Next he should find $I_1(s,q) - I_2(s,q)$. Finally, he should find $I_2(s,q)$.

As the student proceeds in finding the appropriate part of the model, he should use the Long Term Averages option of Program ISL-3 to check his results. To do so he'll have to assume numerical values for w, u, x_{max}, P(x) (x=0,...,x_{max}), c_1, c_2, and c_3. He'll then have to prepare the corresponding parameters for the data in Lines 9010 and 9020. He should next compute I_1, I_2, and I_3, for several sets of suitable values of s and q. He will then be ready to use ISL-3 to check whether his answers agree with those provided by the program.

The student may also wish to use the Simulation option of the program to help in understanding the behavior of the system. For example, he may wish to examine the probability distribution of the amounts on hand at the beginning of each reviewing period.

This manual contains the detailed listing of Program ISL-3. Naturally, the model of the system can be inferred from this listing. It is hoped that the student will refrain from doing so. He should attempt to build the model only through his understanding of the inventory system. Program ISL-3 should only be used to check numerical results, in the same manner that a physicist checks a model of a physical system when he performs an experiment.

5.2 The Solution of the Model

The problem of finding the values of s and q which minimize the expected total cost of the system C(s,q) is an optimization problem. The student is required to develop an algorithm which will yield the optimal values s_o and q_o. This algorithm is not part of Program ISL-3. The only thing the program can do is display the costs in the neighborhood of some specified s and q, as in Examples 4.12, 4.13, and 4.14.

It should be pointed out that the function C(s,q) should not be assumed to be convex. Special precaution must be taken to eliminate the possibility of assuming that a local minimum is the global minimum (e.g., compare the expected total cost tables of Examples 4.13 and 4.14).

Insights into the optimization problem may be gained by studying, in order, Models A to E:

Model A. The units of shortage cost and replenishing cost are relatively very large compared to the unit carrying cost.

Model B. The unit of shortage cost is relatively large compared to the units of carrying cost and replenishing cost.

Model C. The lot size is fixed. An algorithm is required to find only the optimal reorder point s_o.

Model D. An algorithm is required for finding a local minimum of C(s,q).

Model E. An algorithm is required for finding the global minimum of C(s,q).

6. Extensions and Problems for Solution

6.1 Extensions

Program ISL-3 may be extended by relaxing or changing some of the properties of the inventory system:

A. Instead of zero leadtime one may have a leadtime of L periods. L may be a constant or a variable with a probability distribution G(L).

B. The backorder assumption is replaced by the lost sales assumption. That is, shortages are not made up and instead of c_2 being in dollars per unit quantity per unit time, it is in dollars per unit quantity.

C. The reviewing period w is a variable subject to control.

D. A replenishing cost of c_3 is incurred for each lot size (instead of one cost for each replenishment - see Page 15).

6.2 Problems for Solution

A. In an inventory system with an (s,q) policy, no shortages are allowed. The amounts in inventory are reviewed every 2 weeks. The probability distribution of demand during a 2-week period is given by P(0) = 0.25, P(5) = 0.20, P(10) = 0.10, P(15) = 0.20, and P(20) = 0.25. The carrying cost is \$3.20 per unit per week. The replenishing cost is \$180.00.

Assuming that the lot size must be a multiple of 5 units, find the optimal lot size and the corresponding expected minimum total cost of the system.

B. In a probabilistic lot size system with no shortages, the probability density of demand during a reviewing period w is $f(x) = 6x(1-x)$, $0 \le x \le 1$.

Find the optimal lot size as a function of $A = c_3/c_1w$.

C. In a reorder-point system the prescribed lot size is 6 units. The reviewing period is 1 week. The probability distribution of demand during the reviewing period is $P(0) = 0.05$, $P(2) = 0.24$, $P(4) = 0.38$, $P(6) = 0.21$, and $P(8) = 0.12$. The unit carrying cost is $c_1 = \$5$ per week. Find the optimal reorder point for three possible unit costs of shortage $c_2 = \$5$, $c_2 = \$50$, and $c_2 = \$100$ per week.

D. The probability density of demand during the reviewing period is $f(x) = 6x(1 - x)$ $0 \le x \le 1$. An (s,q) policy is employed. The numerical values of the units of carrying cost and shortage cost are equal. What are the optimal reorder points for lot sizes of 0.1 and 2.0?

E. Is the solution found in Example 4.14 the optimal solution?

F. Solve an extension of Problem D when the reviewing period is 2 weeks, the carrying cost is \$5 per unit per week, the shortage cost is \$5 per unit per week, and the replenishing cost is \$20.

G. Solve Problem B when $f(x) = 12(x - 1/2)^2$, $0 \le x \le 1$.

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H. Find the solution of the probabilistic reorder point - lot size system when $w = 1$, $P(0) = 0.08$, $P(10) = 0.10$, $P(20) = 0.20$, $P(30) = 0.30$, $P(40) = 0.16$, $P(50) = 0.10$, and $P(60) = 0.06$, $c_1 = 1$, $c_2 = 10$, and $c_3 = 25$. Show that the minimum expected cost of the system is 49.35.

I. Solve an extension of Problem A when the unit cost of shortage is $50 per week.

7.Program ISL-3

```
ISL-3*


3000REM BY E.NADDOR AND I.PRESSMAN
3010 DIM P(50),F(50),Q(50),G(50),W(50),N(50),V(50)
3020MATP=ZER(50)
3030MATQ=ZER(50)
3040 PRINT"  THE PROBABILISTIC REORDER POINT- LOT SIZE SYSTEM"
3050 PRINT
3060READC1,C2,C3
3070PRINT"CARRYING COST ="C1;"PER UNIT PER PERIOD"
3080PRINT"SHORTAGE COST ="C2;"PER UNIT PER PERIOD"
3090PRINT"REPLENISHING COST ="C3;"PER REPLENISHMENT"
3100LETW1=1
3110 PRINT
3120 READ U,X2
3130 LET J2=INT(X2/U+.01)
3135LETX5=0
3140 FOR J =0 TO J2
3150 READ P(J)
3155LETX5=X5+J*U*P(J)
3160 NEXT J
3170 LET J=J2
3180 LET A=0
3190 LET B=P(J)/J
3200 LET Q(J)=A+B/2
3210 LET A=A+B
3220 LET J=J-1
3230 IF J=0 THEN 3250
3240 GOTO 3190
3250 LET Q(0)=P(0)+A/2
3260PRINT"DEMAND","PROBABILITY","CUMULATIVE"
3270 LET V(0)=P(0)
3280 LET G(0)=Q(0)
3290 LET W(0)=G(0)
3300 LET F(0)=P(0)
3310 LET N(0)=W(0)
3320FORJ=0TO49
3330 LET F(J+1)=F(J)+P(J+1)
3340 LET V(J+1)=V(J)+F(J+1)
3350 LET G(J+1)=G(J)+Q(J+1)
3360 LET W(J+1)= W(J)+G(J+1)
3365IFJ>J2THEN3380
3370PRINTJ*U,P(J),F(J)
3380 LETN(J+1)=N(J)+W(J+1)
3390 NEXT J
4000 PRINT
4010 PRINT"*****************************************************************"
4020 PRINT
4030PRINT"POINT,LOTSIZE,SIM,INIT,RUN,PRINTS,AVER,COSTS"
4040INPUT S1,Q,Y,I0,M,T,S,R
4045PRINT
4046PRINT
```

ISL-3* CONTINUED

```
4047PRINT"REORDER POINT ="S1;"LOT SIZE ="Q
4048PRINT
4049PRINT
4050IFY=1THEN5000
4060IFS=1THEN6000
4070IFR=1THEN7000
4080GOTO4000
5000 PRINT
5010 PRINT
5020 PRINT"                          SIMULATION"
5030 PRINT
5080IFT<=0THEN5120
5090 PRINT
5100 PRINT"PER   BEGIN DEMAND END  ","CARRYING","SHORTAGES   REPLENISHMS
5110 PRINT
5120 LET B1=I0
5130 LET L1=0
5140 LET L2=0
5150 LET L3=0
5155LETL6=0
5160 LET K=0
5170 FOR J= 1 TO M
5180 LET Y=RDM(0)
5190 LETJ9=0
5200 IF F(J9)>Y THEN 5230
5210 LET J9=J9+1
5220 GOTO 5200
5230 LET X = J9*U
5240 LET E1= B1-X
5250  IF B1<=0 THEN 5300
5260 IF E1<0 THEN 5330
5270 LET I1 =(B1+E1)/2
5280 LET I2=0
5290 GOTO 5350
5300 LET I2= (-E1-B1)/2
5310 LET I1=0
5320GOTO5350
5330 LET I1=B1↑2/(2*(B1-E1))
5340 LET I2=E1↑2/(2*(B1-E1))
5350 IF E1 > S1 THEN 5410
5360LETI3=1
5370 LET E2=E1
5380 LET E2=E2+Q
5390 IF E2>S1 THEN 5430
5400 GOTO 5380
5410 LETI3=0
5420 LET E2=E1
5430 IF J>T THEN 5450
5440 PRINT J;B1;X;E1,I1,I2,I3
5450 LET L1=L1+I1
```

ISL-3* CONTINUED

```
5460 LET L2=L2+I2
5470 LET L3=L3+I3
5475LETL6=L6+X
5480 LET B1=E2
5490 NEXT J
5500 PRINT
5510 PRINT "FOR A SIMULATION OF"M;" PERIODS:"
5520LETI1=L1/M
5530LETI2=L2/M
5540LETI3=L3/M
5545LETX6=L6/M
5550GOSUB3500
5560GOTO4060
6000GOSUB3000
6010 PRINT
6020 PRINT
6030 PRINT"                LONG TERM AVERAGES"
6035LETX6=X5
6040GOSUB3500
6070GOTO4070
7000 PRINT
7010 PRINT
7020 PRINT"                THE EXPECTED TOTAL COST TABLE"
7030 PRINT
7040PRINT"   LOT SIZE =",Q-U,Q,Q+U
7050PRINT"R.POINT"
7060 LET S1=S1-2*U
7070 LET Q=Q+U
7080 FOR I=1 TO 3
7090 LET S1=S1+U
7100 PRINT S1,
7110 LET Q=Q-3*U
7120 FOR J=1 TO 3
7130 LET Q=Q+U
7140 GOSUB 8000
7150 PRINT C,
7160 NEXT J
7170 PRINT
7180 NEXT I
7390GOTO4000
8000 LET B= INT((S1+Q-U)/U+.01)
8010IFB>=0THEN8060
8020LETI1=0
8030GOTO8150
8060 LET A= INT((S1-U)/U+.01)
8070 IF A<0 THEN 8100
8080 LET I1=((U↑2)/Q)*(N(B)-N(A))
8090GOTO8150
8100 LET I1=((U↑2)/Q)*N(B)
8150 LET I2=I1+X5/2-((Q+U)/2)-S1
```

ISL-3* CONTINUED

```
8152 IF I2> 1 E-6 THEN 8160
8154 LET I2=0
8160 LET D= INT((Q-U)/U+.01)
8200 LET I3=(1-(U/Q)*V(D))/W1
8210 LET C=C1*I1+C2*I2+C3*I3
8220RETURN
8500PRINT
8502PRINT
8505PRINT" ","DEMAND","CARRYING","SHORTAGES   REPLENISHMENTS"
8510PRINT
8520PRINT"AVERAGES",X6,I1,I2,I3
8530PRINT"UNIT COSTS"," ",C1,C2,C3
8540PRINT"COSTS PER PERIOD ",C1*I1,C2*I2,C3*I3
8550PRINT
8560PRINT"TOTAL COST PER PERIOD ="C1*I1+C2*I2+C3*I3
8570RETURN
9000 REM DATA <C1,C2,C3,U,X2,P(0),P(U),...P(X2)>
9010 DATA 5,50,40
9020 DATA 2,8, .05, .24, .38, .21, .12
9999END
```

INVENTORY SYSTEMS LABORATORY FOUR (ISL-4)

MANUAL 4

by

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Supported by the Office of Education
Contract OEC2-7-070015-3111
and by DHEW, PHS Grant No. HM-00279-01

1. Introduction

This laboratory has been designed for the analysis of a variety of inventory systems. In particular it is suitable for

A. Construction of mathematical models.

B. Checking of Decision Rules.

C. Study of sensitivity of costs to parameters and/or decision rules.

D. Comparing methods of generation of random demands and their use.

E. Analysis of complex inventory systems for which useful mathematical models cannot be constructed.

2. Properties and Policies of the Available Systems

2.1 Demand

Demand during a reviewing period may be deterministic or probabilistic. It occurs uniformily over the period. The characteristics of the demand are supplied as data.

2.2 Replenishments

Replenishments occur only at the beginning of reviewing periods. Quantities ordered are always delivered after the lapse of the appropriate leadtime.

Leadtime may be deterministic or probabilistic. Its characteristics are supplied as data.

2.3 Costs

Four types of costs can be balanced: carrying, shortage, replenishing, and reviewing. Information needed to determine the unit costs is supplied as data.

Carrying costs are based on the unit cost of the item and on the percentage of annual carrying cost. The unit is in [$]/[Q][T].

Shortage costs are either of the back order type (in [$]/[Q][T]) or of the lost sales type (in [$]/[Q]).

The replenishing cost is in [$]. It does not depend on the amount replenished.

The reviewing cost is in [$]. It is incurred every reviewing period.

2.4 Policies

Three inventory policies may be used. The type of policy and its parameters are supplied as inputs during the execution of the program. Various policies and parameters may be examined for each set of demand, leadtime, and costs data.

Policy 1 is the reorder point - lot size system. Policy 2 is the reorder point - order level system. Policy 3 is the scheduling period - order level system. For each policy one has to specify when inventory is ordered (reorder point or scheduling period) and how much is to be ordered (lot size or order level).

The reviewing period is also subject to control and is the third parameter which is supplied for each policy.

2.5 Simulation

A. The number of periods to be simulated is supplied as input during the execution of the program. The number of periods for which details are to be

B. Demands are simulated over a specified number of periods so that their distribution is equal to the distribution of demand supplied as data. The identical demands are used for each simulation. The number of periods is supplied as data.

3. Presentation of Data and Inputs

3.1 Cost Data: T9,C9,P9,H,C2,C3,C4

A. The Period T9

The basic time unit is the period T9. It is measured in days. For example, T9 = 7 days. In this case, then, the period is one week.

B. The Unit Carrying Cost: C1

The unit cost of the item is C9; e.g., C9 = $1285.714 per ton. The annual percentage of carrying cost is P9; e.g., P9 = 20%. The unit carrying cost, C1, is therefore equal to T9 * C9 * P9/360 * 100. For example, C1 = 7 * 1285.714 * 20/36000 = $5 per ton per week.

The program assumes that the year is composed of 360 days and that C1 is measured in [$]/[Q] per period.

C. The Unit Shortage Cost: C2

Two cases are distinguished through the index H: shortages are made up (the back order case), H = 1; and shortages are not made up (the lost sales case), H=2. If H=1 then C2 is in [$]/[Q] per period, e.g. $50 per ton per week.

If H=2, C2 is in [$]/[Q], e.g., $50 per ton.

D. Replenishing and Reviewing Costs: C3, C4

The replenishing cost is C3 per replenishment; e.g., C3 = $40. The reviewing cost C4 is incurred every reviewing period W. E.g., if C4 = $2 and W = 3 weeks, then a reviewing cost of $2 will be incurred every 3 weeks.

E. The Cost Data Statement

The cost data is presented in Line 9010:

```
9010 DATA < T9,C9,P9,H,C2,C3,C4 >
```

For example,

```
9010 DATA 7, 1285.714,20,1,50,40,0
```

3.2 Leadtime Data: 1,L or 2,L1,L,L3,F(L1), F(L1+L3),...,F(L)

The index B distinguishes between deterministic leadtime (B=1) and probabilistic leadtime (B=2). When B=1 then leadtime is L periods; e.g., L=0 weeks. When B=2, the minimum leadtime is L1, the maximum leadtime is L, and the leadtime step is L3. The cumulative distribution of leadtime is given by the non-decreasing non-negative sequence F(L1), F(L1+L3),...,F(L-L3), F(L) where F(L) = 1. For example, if leadtime is 1,3,5, or 7 weeks with equal probabilities, then B=1, L1=1, L=7, L3=2, F(1)=0.25, F(3)=0.50, F(5)=0.75, and F(7)=1.

For deterministic leadtime the data statement is:

```
9020 DATA < 1,L >
```

For example,

```
9020 DATA 1,0
```

85

For probabilistic Leadtime the data statement is

```
9020 DATA < 2,L1,L,L3,F(L1),F(L1+L3),...,F(L) >
```

For example,

```
9020 DATA 2,1,7,2,0.25,0.50,0.75,1.
```

3.3 Demand Data: 1,X(1),X(2),..., or 2,X1,X2,X3,G(X1),G(X1+X3),...,G(X2)

The index C distinguishes between deterministic demand (C=1) and probabilistic demand (C=2). When C=1, the demands are X(1), X(2),...,X(N4). For example X(1)=5, X(2)=1, X(3)=4. (In this case N4=3, but this value does not have to be supplied.) The demands 5,1,4,5,1,4,5,... are used in the simulation.

When C=2, the minimum demand is S1, the maximum demand is X2, and the demand step is X3. The cumulative distribution of demand is G(X1), G(X1+X3),...,G(X2-X3), G(X2)=1. For example, if the probability distribution of demand is P(0)=0.05, P(2)=0.24, P(4)=0.38, P(6)=0.21, and P(8)=0.12, then C=2, X1=0, X2=8, X3=2, G(0)=.15, G(2)=.29, G(4)=.67, G(6)=.88, and G(8)=1.

For deterministic demand the data statement is

```
9030 DATA < 1,X(1),X(2),... >
```

For example,

```
9030 DATA < 1,5,1,4 >
```

For probabilistic demand the data statement is

```
9030 DATA < 2,X1,X2,X3,G(X1),G(X1+X3),...,G(X2) >
```

For example,

```
9030 DATA 2,0,8,2,.05,.29,.67,.88,1
```

3.4. Demands Cycle: N4

When demand is deterministic (C=1) the demands cycle every N4 periods, N4 does not have to be specified.

When demand is probabilistic (C=2), demands also cycle every N4 periods. The program generates random demands so that their distribution over N4 periods will equal the given distribution G(X1),...,G(X2). The number of periods N4 should be selected accordingly. It is given in the data statement

```
9010 DATA < N4 >
```

For example,

```
9010 DATA 100
```

3.5 Summary of Data Statements

The general format of the data statements is:

```
9000 DATA < N4 >
9010 DATA < T9,C9,P9,H,C2,C3,C4 >
9020 DATA < 1,L >  or
9020 DATA < 2,L1,L,L3,F(L1),F(L1+L3),...,F(L)=1 >
9030 DATA < 1,X(1),X(2),... >  or
9030 DATA < 2,X1,X2,X3,G(X1),G(X1+X3),...,G(X2)=1 >
```

For example,

```
9000 DATA 100
9010 DATA 7,1285.714,20,1,50,40,0
9020 DATA 1,0  or
9020 DATA 2,1,7,2,0.25,0.50,0.75,1
9030 DATA 1,5,1,4  or
9030 DATA 2,0,8,2,.05,.29,.67,.88,1
```

3.6 Policy Inputs: POLICY,WHEN,HOW MUCH,REVIEW

During the execution of the program, the user has to supply the policy, J, and its parameters V(1), V(2), and V(3). The policy J can be 1,2, or 3. When J=1 (the reorder point-lot size system) V(1) is the reorder point and V(2) is the lot size. When J=2 (the reorder point-order level system) V(1) is again the reorder point but V(2) is the order level. When J=3 (the scheduling period-order level system) V(1) is the scheduling period and V(2) is the order level. For any policy,V(3) is the reviewing period: the number of basic periods in the reviewing period.

Examples of inputs are:

```
     POLICY,WHEN,HOW MUCH,REVIEW
A.      1  , -2 ,   10   ,  1
B.      2  ,  0 ,   14   ,  2
C.      3  ,  4 ,   10   ,  4
```

In Case A inventory is reviewed every period. When the amount on hand and on order is -2 or below, an order is placed for 10 units.

In Case B inventory is reviewed every 2 weeks. When the amount on hand and on order is zero, or below an order is placed to raise this amount to 14 units.

In Case C the amount on hand and on order is reviewed every 4 periods. Orders are then placed so as to raise the amount to the level of 10 units.

3.7 Simulation Inputs: SIM RUN,PRINTS

When the user supplies the policy inputs he also has to supply the size of the simulation run, M, and the number of periods, N, for which detailed information should be printed. Obviously, $N \leq M$. When N=0 no detailed printouts are given, only the summary of results is printed. When N=1, only a brief summary is given. For example,

	SIM RUN	,	PRINTS
A.	10	,	10
B.	100	,	0
C.	500	,	-1

In Case A, 10 periods will be simulated and details for each period will be printed. In Case B, 100 periods will be simulated, no details for individual periods will be given, but a standard summary of results will be printed. In Case C, 500 periodswill be simulated and only a brief summary will be printed.

3.8 Optimization Inputs: ADV VARIABLE,STEP

Together with the policy and simulation inputs the user has to supply 2 optimization inputs. The first optimization input, O, is a policy variable and can be 0,1,2, or 3. When O=0 then the optimization option is ignored. Otherwise, three consecutive simulations are executed. The same input parameters are used in every simulation with the exception of the policy variable O.

The optimization input, X3, is a step for the variable 0. In the three consecutive simulations the variable assumes the values V(0),V(0) + X3, and V(0) + X3 + X3 respectively. This capability is useful for checking optimal solutions.

Examples of optimization inputs are:

```
      ADV VARIABLE,STEP
  A.       0      , 0
  B.       3      , 1
  C.       2      , 5
```

In Case A, the optimization option is ignored, and only one simulation will be given. (The step, too, is ignored.) In Case B, three simulations will be given for V(3), V(3) + 1, and V(3) + 2. In Case C, the three simulations will be for V(2), V(2) + 5, and V(2) + 10.

3.9. Summary of Inputs

The general format for request of inputs is:

```
POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP
? < J ,V(1),  V(2)  , V(3) ,   M   ,  N  ,     0      , X3 >
```

Examples of inputs are:

```
?  1 ,  -2,   10   ,  1 ,  10   , 10  ,     0      , 0    (A.)
?  2 ,   0,   14   ,  2 , 100   ,  0  ,     3      , 1    (B.)
?  3 ,   4,   10   ,  4 , 500   , -1  ,     2      , 5    (C.)
```

In Case A the reorder point is -2, the lot size is 10, and the reviewing period is 1. There will be 10 periods in the simulation and the details of each of the 10 periods will be printed. No variable will be advanced - only one simulation will be executed.

In Case B the reorder point is 0, the order level is 14, and the first reviewing period is 2. There will be 100 periods in each simulation and only the summary of results will be printed. In the second simulation the reviewing period will be 3, and in the third it will be 4.

In Case C the scheduling period and the reviewing period are both 4, and the first order level is 10. Each simulation will have 500 periods and only brief summaries will be printed. In the second simulation the order level will be 15, and in the third it will be 20.

4. Illustrations

Two inventory systems are treated in this section. They illustrate all the capabilities of Program ISL-4 and they provide examples of actual computer outputs.

4.1 Illustration 1

In an inventory system the weekly demands are in units of 2 pounds and have the probabilities: P(0)=.05, P(2)=.24, P(4)=.38, P(6)=.21, and P(8)=.12. Leadtime is zero. The cost of one pound is $1285.714, and the annual carrying cost is 20% of this cost. Shortages can be made up, but there is a penalty cost of $50 per pound per week. The ordering cost is $40.

Simulate the system using three different policies.

The data, the computer output, the three inputs, and the results are given below:

```
9000 DATA 100
9010 DATA 7, 1285.714, 20, 1, 50, 40, 0
9020 DATA 1, 0
9030 DATA 2, 0, 8, 2, .05, .29, .67, .88, 1
KEY
READY.

RUN
WAIT.


ISL-4     16:08   W9 FRI 09/29/67


INVENTORY SYSTEMS SIMULATION, ONE PERIOD= 7    DAYS

UNIT COST= 1285.714 PERCENT/YEAR= 20   CARRYING COST/UNIT/PERIOD= 5.

SHORTAGES MADE UP   SHORTAGE COST/UNIT/PERIOD= 50

REPLENISHING COST= 40   REVIEWING COST= 0

LEADTIME= 0     PERIODS


DEMAND     PROBABILITY

 0              .05
 2              .24
 4              .38
 6              .21
 8              .12

IN 100 PERIODS: AVERAGE DEMAND = 4.22    STANDARD DEVIATION = 2.11462
```

```
***********************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP
? 1  , -2 ,  8   , 1  ,  5  ,  5  ,     0     , 0

 -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -

REORDER POINT =-2    LOT SIZE = 8    REVIEWING PERIOD = 1

PER    BEG    DEM    END    AVA    ORD    REC    CAR    SHO    REP    REV
                                                  ROUNDED OFF

 1      6      6      0      0      0      0      3      0      0      1
 2      0      2     -2     -2      8      8      0      1      1      1
 3      6      2      4      4      0      0      5      0      0      1
 4      4      4      0      0      0      0      2      0      0      1
 5      0      2     -2     -2      8      8      0      1      1      1

FOR A SIMULATION OF  5   PERIODS:

                  CARRYING          SHORTAGE          REPLENISH         REVIEW

AVERAGES          2                 .4                .4                1
UNIT COSTS        5.                50                40                0
COSTS             10.               20.               16.               0

TOTAL COST PER PERIOD= 46.
```

Example 4.11

```
***********************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP
? 2  ,  0 ,  10   , 1  , 100  , 8  ,     0     , 0

 -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -

REORDER POINT = 0    ORDER LEVEL = 10   REVIEWING PERIOD = 1

PER    BEG    DEM    END    AVA    ORD    REC    CAR    SHO    REP    REV
                                                  ROUNDED OFF

 1     10      6      4      4      0      0      7      0      0      1
 2      4      2      2      2      0      0      3      0      0      1
 3      2      2      0      0     10     10      1      0      1      1
 4     10      4      6      6      0      0      8      0      0      1
 5      6      2      4      4      0      0      5      0      0      1
 6      4      4      0      0     10     10      2      0      1      1
 7     10      2      8      8      0      0      9      0      0      1
 8      8      8      0      0     10     10      4      0      1      1

FOR A SIMULATION OF  100  PERIODS:

                  CARRYING          SHORTAGE          REPLENISH         REVIEW

AVERAGES          4.65667           .146667           .37               1
UNIT COSTS        5.                50                40                0
COSTS             23.2833           7.33333           14.8              0

TOTAL COST PER PERIOD= 45.4167
```

Example 4.12

Example 4.13

```
**************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP
? 3 , 3 , 12  , 2 , 600 , 7 ,  0  , 0

 -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -

SCHEDULING PERIOD = 3   ORDER LEVEL = 12   REVIEWING PERIOD = 2

PER   BEG   DEM   END   AVA.  ORD   REC   CAR    SHO   REP   REV
                                        ROUNDED OFF

 1    12    6     6     6     0     0     9      0     0     0
 2    6     2     4     4     0     0     5      0     0     0
 3    4     2     2     2     10    10    3      0     1     1
 4    12    4     8     8     0     0     10     0     0     0
 5    8     2     6     6     0     0     7      0     0     0
 6    6     4     2     2     10    10    4      0     1     1
 7    12    2     10    10    0     0     11     0     0     0

FOR A SIMULATION OF  600  PERIODS:

                  CARRYING        SHORTAGE       REPLENISH      REVIEW

AVERAGES          5.985           .315           .333333        .333333
UNIT COSTS        5.              50             40             0
COSTS             29.925          15.75          13.3333        0

TOTAL COST PER PERIOD= 59.0033

**************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP
? STOP

RAN      19 SEC.
```

The following points should be noted:

a. Line 9000 on Page 12 assures that 100 random demands are generated. These demands follow the given distribution and are used in the identical order in Examples 4.11, 4.12, and 4.13.

b. In Line 9010 on Page 12, H=1 assures that shortages are made up and that the reviewing cost is zero.

c. Since the generated demands follow precisely the given distribution the mean and the standard deviation on Page 12 are also the mean and deviation of the distribution.

d. The abbreviations in Examples 4.11, 4.12, and 4.13 are:

PER = period, BEG = begin, DEM = demand, END = end, AVA = available,

ORD = order, REC = receive, CAR = carrying, SHO = shortage,

REP = replenishment, REV = review.

e. In Example 4.11, the average amounts carried in the four weeks are 3,0,5,2, and 0 pounds. Hence, the overall average is (3+0+5+2+0)/5 = 2 pounds, as can be seen in the summary of the results. In a similar way other averages are computed.

f. In Example 4.12 no shortages occur during the first 8 weeks. However, some shortages occur during the remaining 92 weeks, so that the overall average is .146667 pounds.

g. In Example 4.13 the input specifies a scheduling period of 3 weeks, and a reviewing period of 2 weeks. The program disregards the value of the reviewing period whenever the third policy (J=3) is used. It always assumes that in this case the reviewing period and the scheduling periods are one and the same.

4.2 Illustration 2

In an inventory system demands cycle every 7 weeks in the following order: 4,2,6,4,8,2, and 4 tons. Leadtime can be zero, one, or two weeks with respective probabilities of .2, .6, and .2. The cost of one ton is $1285.714, and the annual carrying cost is 20% of this cost. Whenever demand cannot be satisfied, there is a loss of $50 per ton. The replenishing cost is $40 and the reviewing cost is $1.5.

Simulate the system using three different policies. For Policy 1 detailed printouts are desired for 15 weeks. For Policy 2 only a summary is desired, and for Policy 3 only a brief summary is desired.

The data, output, input, and results are given in Pages 17 and 18.

The following points should be noted:

a. The demands in all examples follow the pattern illustrated in Example 4.21.

b. During the 10th week, in Example 4.21, the correct average amount in inventory is $4^2/2x6 = 1.333$. Only the rounded off value is given in the table. However, the correct amount is used for the summary.

c. The main reason for the increase in shortages in Example 4.22 as compared with Example 4.21 is the change in the reviewing period.

d. The abbreviations in the brief summary in Example 4.23 correspond to those listed earlier. TOT stands for 'total'. All the numerical values are costs per period.

e. For an explanation of the 1.5 cost of reviewing in Example 4.23, see note g. of the previous section.

```
READY.


9000 DATA 10
9010 DATA 7, 1235.714, 20, 2, 50, 40, 1.5
9020 DATA 2, 0, 2, 1, .2, .8, 1
9030 DATA 1, 4, 2, 6, 4, 8, 2, 4
KEY
READY.

RUN
WAIT.



ISL-4     16:22   N2 FRI 09/29/67


INVENTORY SYSTEMS SIMULATION, ONE PERIOD= 7    DAYS

UNIT COST= 1285.71 PERCENT/YEAR= 20   CARRYING COST/UNIT/PERIOD= 5.

SHORTAGES NOT MADE UP.  SHORTAGE COST/UNIT= 50

REPLENISHING COST= 40   REVIEWING COST= 1.5

LEADTIME         PROBABILITY

 0                   .2
 1                   .6
 2                   .2


DEMAND       4     2      6      4      8      2      4
IN 7     PERIODS: AVERAGE DEMAND = 4.28571 STANDARD DEVIATION = 1.97949

*************************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP
? 1,   4,    8,      1,      15,      15,         0,      0
```

Example 4.21

```
 -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -

REORDER POINT = 4    LOT SIZE = 8    REVIEWING PERIOD = 1

PER    BEG    DEM    END    AVA    ORD    REC    CAR    SHO    REP    REV
                                                ROUNDED OFF

 1     12      4      8      8      0      0      10     0      0      1
 2      8      2      6      6      0      0      7      0      0      1
 3      6      6      0      0      8      0      3      0      1      1
 4      0      4      0      8      0      8      0      4      0      1
 5      8      8      0      0      8      8      4      0      1      1
 6   [illegible] 2    6      6      0      0      7      0      0      1
 7   [illegible] 4    2      2      8      8      4      0      1      1
 8     10      4      6      6      0      0      8      0      0      1
 9      6      2      4      4      8      0      5      0      1      1
 10     4      6      0      8      0      8      1      2      0      1
 11     8      4      4      4      8      0      6      0      1      1
 12     4      8      0      8      0      8      1      4      0      1
 13     8      2      6      6      0      0      7      0      0      1
 14     6      4      2      2      8      0      4      0      1      1
 15     2      4      0      8      0      8      1      2      0      1
```

```
FOR A SIMULATION OF  15   PERIODS:

                     CARRYING         SHORTAGE          REPLENISH       REVIEW

AVERAGES             4.05556           .8               .466667         1
UNIT COSTS           5.                50               40              1.5
COSTS                20.2778           40.              18.6667         1.5

TOTAL COST PER PERIOD= 80.4444

************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP     Example 4.22
?  2,   4,     12,       2,      700,      0,          0,          0

  -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -

REORDER POINT = 4     ORDER LEVEL = 12    REVIEWING PERIOD = 2

FOR A SIMULATION OF  700  PERIODS:

                     CARRYING         SHORTAGE          REPLENISH       REVIEW

AVERAGES             4.3494           1.18286           .288571         .5
UNIT COSTS           5.               50                40              1.5
COSTS                21.747           59.1429           11.5429         .75

TOTAL COST PER PERIOD= 93.1827

************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP    Example 4.23
?  3,   2,     12,          1,     70,      -1,          0,          0

  -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -

SCHEDULING PERIOD = 2     ORDER LEVEL = 12    REVIEWING PERIOD = 1

CAR= 22.8988   SHO= 41.4286   REP= 19.4286   REV= .75         TOT= 84.5059

************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP
? STOP

RAN     26 SEC.


STOP.
READY.
```

5. Optimization

Comments on the construction of models and on the methods of finding optimal solutions are given in Manuals 2 and 3. In this section we only illustrate the use of the optimization feature of Program ISL-4.

We consider Illustration 1 of Section 4.1. Suppose one is interested in Policy 1, and it is conjectured that the optimal reorder point is 0 and the optimal lot size is 8. The data, outputs, inputs, and results are as on Pages 20 and 21.

The inputs in Example 5.1 have been selected so that the lot size is fixed at 8, and three reorder points are used: 0,2, and 4, the step being 2. One notes that the total cost for a reorder point of 2 (48.7258) is lower than the costs for reorder points of 0 and 4 (48.9925 and 55.9575). Similarly, in Example 5.2 the reorder point is fixed at 2. The cost for a lot size of 8 (48.7258) is lower than the costs for lot sizes of 6 and 10 (49.8742 and 48.75). A further check (Example 5.3) indicates that for fixed reorder point of 2 and lot size of 8, a reviewing period of 1 week has the lowest cost (48.7258). It may thus appear that the conjecture is true. Unfortunately, this is not the optimal solution, as can be seen from Example 5.4.

Incidentally, the long range expected costs can be shown to be:

Lot Size	6	8	10	12
Reorder Point				
-2	81.717	67.762	60.100	56.658
0	53.583	48.750	46.890	47.317
2	50.017	48.575	48.750	50.533
4	56.533	55.962	56.660	58.792

```
TAPE
READY.

9000 DATA 100
9010 DATA 7, 1285.714, 20, 1, 50, 40, 0
9020 DATA 1, 0
9030 DATA 2, 0, 8, 2, .05, .29, .67, .88, 1
KEY
READY.

RUN
WAIT.

ISL-4      16:35   #2 FRI 09/29/67

INVENTORY SYSTEMS SIMULATION, ONE PERIOD= 7    DAYS

UNIT COST= 1285.71 PERCENT/YEAR= 20   CARRYING COST/UNIT/PERIOD= 5.

SHORTAGES MADE UP.  SHORTAGE COST/UNIT/PERIOD= 50

REPLENISHING COST= 40   REVIEWING COST= 0

LEADTIME= 0    PERIODS

DEMAND     PROBABILITY

 0              .05
 2              .24
 4              .38
 6              .21
 8              .12

IN 100  PERIODS: AVERAGE DEMAND = 4.22    STANDARD DEVIATION = 2.11462

**************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP
? 1,    0,     8,     1,     500,    -1,      1,         2

 -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -

REORDER POINT = 0    LOT SIZE = 8    REVIEWING PERIOD = 1

CAR= 15.6775   SHO= 12.275    REP= 21.04      REV= 0          TOT= 48.9925

 -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -

REORDER POINT = 2    LOT SIZE = 8    REVIEWING PERIOD = 1

CAR= 24.7442   SHO= 2.94167   REP= 21.04      REV= 0          TOT= 48.7258

 -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -

REORDER POINT = 4    LOT SIZE = 8    REVIEWING PERIOD = 1

CAR= 34.4925   SHO= .425      REP= 21.04      REV= 0          TOT= 55.9575
```

Example 5.1

```
***************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP    Example 5.2
?  1,     2,     6,        1,      500,      -1,       2,          2

 -    -    -    -    -    -    -    -    -    -    -    -    -    -    -    -

REORDER POINT = 2    LOT SIZE = 6    REVIEWING PERIOD = 1

CAR= 19.6558    SHO= 3.65833    REP= 26.56      REV= 0          TOT= 49.8742

 -    -    -    -    -    -    -    -    -    -    -    -    -    -    -    -

REORDER POINT = 2    LOT SIZE = 8    REVIEWING PERIOD = 1

CAR= 24.7442    SHO= 2.94167    REP= 21.04      REV= 0          TOT= 48.7258

 -    -    -    -    -    -    -    -    -    -    -    -    -    -    -    -

REORDER POINT = 2    LOT SIZE = 10   REVIEWING PERIOD = 1

CAR= 29.67      SHO= 2.2        REP= 16.88      REV= 0          TOT= 48.75
***************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP    Example 5.3
 ? 1,    2,    8,        1,      500,      -1,       3,          1

 -    -    -    -    -    -    -    -    -    -    -    -    -    -    -    -

REORDER POINT = 2    LOT SIZE = 8    REVIEWING PERIOD = 1

CAR= 24.7442    SHO= 2.94167    REP= 21.04      REV= 0          TOT= 48.7258

 -    -    -    -    -    -    -    -    -    -    -    -    -    -    -    -

REORDER POINT = 2    LOT SIZE = 8    REVIEWING PERIOD = 2

CAR= 16.7842    SHO= 28.9417    REP= 17.76      REV= 0          TOT= 63.4858

 -    -    -    -    -    -    -    -    -    -    -    -    -    -    -    -

REORDER POINT = 2    LOT SIZE = 8    REVIEWING PERIOD = 3

CAR= 12.3692    SHO= 83.9917    REP= 12.88      REV= 0          TOT= 109.241
```

```
***************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP
?  1,     0,    10,      1,      500,    -1,       0,         0

 -    -    -    -    -    -    -    -    -    -    -    -    -    -    -    -

REORDER POINT = 0     LOT SIZE = 10    REVIEWING PERIOD = 1

CAR= 20.41      SHO= 9.6        REP= 16.88      REV= 0          TOT= 46.89

***************************************************************

POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP
 ? STOP

RAN      75 SEC.
```

Example 5.4

6. Extensions and Problems for Solution

6.1 Extensions

Program ISL-4 is quite a general program and allows for the analysis of a great variety of inventory systems. However, it is still possible to extend the program in many directions. Some of these extensions are stated in the form of exercises.

A. The initial inventory in ISL-4 depends on the policy used. When J=1, the initial inventory is V(1) + V(2), otherwise it is V(2). Examine the adequacy of this initial inventory, especially when leadtime is not zero. Suggest other initial values to reduce the necessity for larger simulation runs.

B. The occurrences of leadtime in a simulation may not necessarily follow the prescribed distribution of leadtime (e.g., Example 4.21). Suggest a more satisfactory method for generating leadtime.

C. The cost of replenishing is independent of the amount replenished. Change the program so that this cost will be some function of this amount.

D. Assume that the cost of reviewing is zero and eliminate all the printing statements related to reviewing. However, leave the REVIEW variable (V(3) in the program and allow its value to be supplied as input.

E. In ISL-4 whenever an amount P is ordered, this amount is actually received after the elapse of the appropriate leadtime. Change the program so that the amount received is a random variable from some distribution one of whose parameters is P.

F. Define a fourth policy (J=4) and allow its use during the execution of the program.

6.2. Problem for Solution

A. In a reorder point - lot size system, the amounts in inventory are reviewed every 2 weeks. The probability density of demand during a 2-week period is given by $f(x) = 6x(1 - x)$, $0 \le x \le 1$. The carrying cost is \$5 per unit per week, the shortage cost is \$5 per unit per week, and the replenishing cost is \$20. Convert the density to a discrete distribution and show that the optimal lot size and reorder point are approximately 2.0 and - 0.8 respectively.

B. Find the solution of a reorder point - lot size system when the reviewing period is one week, the probability distribution of demand during a week is $P(0) = .08$, $P(10) = .10$, $P(20) = .20$, $P(30) = .30$, $P(40) = .16$, $P(50) = .10$, and $P(60) = .06$. The carrying cost is \$1 per unit per week, the shortage cost is \$10 per unit per week, and the replenishing cost is \$25. Show that the minimum expected total cost of the system is \$49.35 per week.

C. The demand during any week is either 0 or 1 with probabilities 0.4 and 0.6 respectively. Leadtime is 3 weeks. The carrying cost and the shortage cost are \$1 per unit per week and \$5 per unit per week respectively. The replenishing cost is \$2. A (t,Z) policy is used. Find the optimal scheduling period, the optimal order level, and the corresponding expected minimum total cost of the system.

D. In a reorder point - lot size system inventories are reviewed every week. Leadtime is 3 weeks. During any week there is a demand for one unit with the probability 0.6. Otherwise there is no demand. The carrying cost is \$1 per unit per week, the shortage cost is \$5 per unit per week, and the replenishing cost is \$2. Find the optimal reorder point, the optimal lot size and the corresponding minimum expected total cost.

E. In an inventory system with a reorder-point-order-level policy the reviewing period is 2 weeks. The probability distribution of demand during the reviewing period is P(0) = 0.20, P(0.5) = 0.24, P(1,0) = 0.40, P(1.5) = 0.16. The carrying cost is \$5 per unit per week. The shortage cost is \$10 per unit per week. The replenishing cost is \$40. What is the optimal reorder point and what is the optimal order level?

F. In an inventory system the reorder point is 2 and the order level is 5. The probability distribution of demand on any day is P(0) = 0.5, P(1) = 0.3, P(2) = 0.1, and P(3) = 0.1.

(1) Show that the probability of a replenishment on any day is 25/108.

(2) By changing Program ISL-4, show that on the average there will be 227/54 units in stock at the beginning of each day.

G. In an inventory system the reviewing period is 1 week. Demand during the reviewing period has the distribution P(0) = 0.2, P(1) = 0.4, P(2) = 0.4. The carrying cost is $1 per unit per week, the shortage cost $1 per unit per week, and replenishing cost $1 per replenishment. Compare the minimum costs of the sytem for each of the three policies.

7. Program ISL-4

```
ISL-4


0REM BY E.NADDOR
1000DIMF(11),G(20),H(20),V(3)
1002DIMX(500)
1005READN4
1010READT9,C9,P9,H,C2,C3,C4
1020LETC1=T9*C9*P9/36000
1030READB
1040IFB=2THEN 1080
1050READL
1070GOTO 1160
1080READL1,L,L3
1100 FOR I = L1 TO L STEP L3
1110 READ F(I)
1130 NEXT I
1160 READ C
1170 IF C = 1 THEN 1230
1180READX1,X2,X3
1185READG(X1)
1190LETH(X1)=INT(G(X1)*N4+.01)
1195FORI=X1+X3TOX2STEPX3
1200READG(I)
1205LETH(I)=INT((G(I)-G(I-X3))*N4+.01)
1210 NEXT I
1220GOTO 1350
1230LETX4=0
1240LETX5=0
1250LETI=1
1260 READ X(I)
1270 IF X(I) = 9999 THEN 1320
1280LETX4=X4+X(I)
1290LETX5=X5+X(I)*X(I)
1300 LET I = I + 1
1310 GOTO 1260
1320 LET N4 = I - 1
1330GOTO 1470
1350LETX4=0
1360LETX5=0
1370FORI=1TON4
1380LETX=X1
1390LETY=RND(0)
1400IFY<G(X)THEN 1430
1410LETX=X+X3
1420GOTO 1400
1430IFH(X)=0THEN 1380
1432LETX(I)=X
1435LETH(X)=H(X)-1
1440LETX4=X4+X
1450LETX5=X5+X*X
1460NEXTI
1470LETX5=SQR((X5-X4*X4/N4)/N4)
```

ISL-4 CONTINUED

```
1480LETX4=X4/N4
2000PRINT
2010PRINT"INVENTORY SYSTEMS SIMULATION, ONE PERIOD="T9;"DAYS"
2020PRINT
2030PRINT"UNIT COST="C9;"PERCENT/YEAR="P9;"CARRYING COST/UNIT/PERIOD="C1
2040PRINT
2050IFH=2THEN 2080
2060PRINT"SHORTAGES MADE UP.  SHORTAGE COST/UNIT/PERIOD="C2
2070GOTO 2090
2080PRINT"SHORTAGES NOT MADE UP.  SHORTAGE COST/UNIT="C2
2090PRINT
2095PRINT"REPLENISHING COST="C3;"REVIEWING COST="C4
2096PRINT
2100IFB=2THEN 2130
2110PRINT"LEADTIME="L;"PERIODS"
2120GOTO 2190
2130PRINT"LEADTIME","PROBABILITY"
2140PRINT
2150PRINTL1,F(L1)
2160FORI=L1+L3TOLSTEPL3
2170PRINTI,F(I)-F(I-L3)
2180NEXTI
2190PRINT
2200PRINT
2210PRINT"DEMAND      ";
2220IFC=2THEN 2270
2230FORI=1TON4
2240PRINTX(I);
2250NEXTI
2260GOTO 2350
2270PRINT"PROBABILITY"
2280PRINT
2290PRINTX1,
2300PRINTG(X1)
2310FORI=X1+X3TOX2STEPX3
2320PRINTI,
2330PRINTG(I)-G(I-X3)
2340NEXTI
2350PRINT
2360PRINT"IN"N4;"PERIODS: AVERAGE DEMAND ="X4;"STANDARD DEVIATION ="X5
3000PRINT
3010PRINT"*************************************************************"
3020PRINT
3030LETE=1
3040PRINT"POLICY,WHEN,HOW MUCH,REVIEW,SIM RUN,PRINTS,ADV VARIABLE,STEP";
3050PRINT
3060INPUTJ,V(1),V(2),V(3),M,N,O,X3
4000PRINT
4010PRINT" -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   "
4020PRINT
```

ISL-4 CONTINUED

```
4030LETZ=V(1)
4040LETQ=V(2)
4050LETW=V(3)
4500LETJ1=0
4510LETJ2=0
4520LETJ3=0
4530LETJ4=0
4540IFJ=1THEN 4580
4550LETQ2=Q
4570GOTO 4610
4580LETQ2=Z+Q
4610FORI=0TOL+1
4620LETR(I)=0
4630NEXTI
5000IFJ=2THEN 5040
5010IFJ=3THEN 5060
5020PRINT"REORDER POINT ="Z;"LOT SIZE ="Q;
5030GOTO 5070
5040PRINT"REORDER POINT ="Z;"ORDER LEVEL ="Q;
5050GOTO 5070
5060PRINT"SCHEDULING PERIOD ="Z;"ORDER LEVEL ="Q;
5070PRINT"REVIEWING PERIOD ="W
6000IFN<=0THEN 7000
6010PRINT
6020PRINT"PER.   BEG    DEM    END    AVA    ORD    REC    CAR    SHO    REP";
6030PRINT"    REV"
6040PRINT"                                                        ROUNDED OFF"
6050PRINT
7000FORK=1TOM+L
7010 LET Q1 = Q2 + R(0)
7020 LET X = X(K - N4*INT((K-0.01)/N4))
7030 LET Q2 = Q1 - X
7100 IF Q2 >= 0 THEN 7150
7110 IF Q1 > 0 THEN 7180
7120 LET I1 = 0
7130 LET I2 = -(Q1 + Q2)/2
7140GOTO 7200
7150 LET I1 = (Q1 + Q2)/2
7160 LET I2 = 0
7170GOTO 7200
7180 LET I2=Q2↑2/(2*X)
7190 LET I1=Q1↑2/(2*X)
7200IFH=1THEN 7540
7205IFQ2>=0THEN 7540
7210LETI2=-Q2
7220LETQ2=0
7520 LET A = 0
7530 GOTO 7550
7540 LET A = Q2
7550 IF L = 0 THEN 7590
```

ISL-4 CONTINUED

```
7560 FOR I = 1 TO L
7570 LET A = A + R(I)
7580 NEXT I
7590 LET P = 0
7600 LET I3 = 0
7605 LET I4=0
7610 IF J = 3 THEN 7680
7620IFK/W=INT(K/W)THEN 7626
7624GOTO 7800
7626LETI4=1
7630 IF A>Z THEN 7800
7640 IF J = 2 THEN 7690
7650 LET P = P + Q
7660IFA+P<=ZTHEN 7650
7670GOTO 7710
7680 IF K/Z<>INT(K/Z) THEN 7800
7685 LET I4=1
7690 LET P = Q - A
7700IFP=0THEN 7800
7710 LET I3 = 1
7720 IF B = 1 THEN 7780
7730 LET I = L1
7740 LET Y = RND(W)
7750IFY<F(I)THEN 7790
7760 LET I = I + L3
7770 GOTO 7750
7780 LET I = L
7790 LET R(I + 1) = R(I + 1) + P
7800 FOR I = 0 TO L
7810 LET R(I) = R(I + 1)
7820 NEXT I
7830 LET R(L+1) = 0
7875IFK<=LTHEN 7920
7880 LET J1 = J1 + I1
7890 LET J2 = J2 + I2
7900 LET J3 = J3 + I3
7910LETJ4=J4+I4
7920 IF K>N THEN 7950
7930PRINTK;Q1;X;Z;A;P;R(0);INT(I1+.5);INT(I2+.5);I3;
7940PRINTI4
7950 NEXT K
8000LETI1=J1/M
8010LETI2=J2/M
8020LETI3=J3/M
8030LETI4=J4/M
8040LETK1=I1*C1
8050LETK2=I2*C2
8060LETK3=I3*C3
8070LETK4=I4*C4
8080LETK0=K1+K2+K3+K4
```

ISL-4 CONTINUED

```
8090PRINT
8100 PRINT
8110IFN<0THEN 8260
8120PRINT "FOR A SIMULATION OF "M;"PERIODS:"
8130 PRINT
8140PRINT" ","CARRYING","SHORTAGE","REPLENISH",
8150PRINT"REVIEW"
8160PRINT
8170PRINT"AVERAGES",I1,I2,I3,
8180PRINTI4
8190PRINT"UNIT COSTS",C1,C2,C3,
8200PRINTC4
8210PRINT"COSTS",K1,K2,K3,
8220PRINTK4
8230PRINT
8240PRINT"TOTAL COST PER PERIOD="K0
8250GOTO 8290
8260PRINT"CAR="K1,"SHO="K2,"REP="K3,
8270PRINT"REV="K4,
8280PRINT"TOT="K0
8290IFO=0THEN 3000
8300IFE=3THEN 3000
8310LETE=E+1
8320LETV(O)=V(O)+X3
8330GOTO 4000
9000DATA100
9010DATA7,1285.714,20,1,50,40,0
9020DATA1,0
9030DATA2,0,8,2,.05,.29,.67,.88,1
9998DATA9999
9999END
```

Appendix E

FORTRAN Listings and Data

Page

ISL-1: Balancing Carrying, Shortage, and Replenishing Costs

Listing 112

Data 113

ISL-2: The Deterministic Reorder Point--Lot Size System

Listing 115

Data 117

ISL-3: The Probabilistic Reorder Point--Lot Size System

Listing 119

Data 123

ISL-4: A General Inventory Systems Simulation

Listing 125

Data 131

ISL-1: Balancing Carrying, Shortage, and Replenishing Costs

```
C     ISL-1 BY E.NADDOR AND OLE BRAATEN
C     FORTRAN VERSION BY RICHARD SACHER
C     THE JOHNS HOPKINS UNIVERSITY, BALTIMORE MD. 21218
C
C     BASED ON MANUAL ISL-1, SEPTEMBER, 1967
C
      DIMENSION D(50),E(50),R(50),B(50)
      REAL I1, I2, I3, I11, I21, I31
C
C     INITIALIZATION
C
   15 DEMAND=0.
      REPLEN=0.
      I1=0.
      I2=0.
      I3=0.
      DO 20  I=1,60
   20 R(I)=0.
C
C     READING OF DATA (N=PERIODS,D(I)=DEMAND,C1,C2,C3=UNIT COSTS)
C
      READ(5,1) N
      READ(5,2) (D(I), I=1,N)
      READ (5,2) C1, C2, C3
C
C     READING OF DECISIONS DATA* B(1)=BEGIN, I,R(I)=PERIOD AND REPLENISHMENT
C
      READ(5,2) B(1)
      XN = N
      DO 125  I=1,N
  125 DEMAND=DEMAND + D(I)
  130 READ(5,8) I, R(I)
      IF (I) 150,100,100
  100 REPLEN = REPLEN + R(I)
      GO TO 130
  150 IF (REPLEN - DEMAND) 999,200,999
C
C     MAIN PROGRAM
C
  200 DO 285  I=1,N
      E(I)=B(I)+R(I)-D(I)
      IF (E(I)) 240,250,250
  240 I2 = I2 - E(I)
      GO TO 270
  250 I1 = I1 + E(I)
  270 IF (R(I)) 285,285,280
  280 I3=I3 + 1.
  285 B(I+1)=E(I)
C
C     AVERAGES
C
      I11=I1/XN
      I21=I2/XN
      I31=I3/XN
      COST1=I11*C1
      COST2=I21*C2
      COST3=I31*C3
      COST4=COST1 + COST2 + COST3
      COST11=COST1*XN
      COST21=COST2*XN
      COST31=COST3*XN
      COST41 = COST11 + COST21 + COST31
      WRITE(6,3)
      WRITE(6,4) (I,B(I),R(I),D(I),E(I),  I= 1,N)
      WRITE(6,5)
      WRITE(6,6) I11,I21,I31,C1,C2,C3
      WRITE(6,7) COST1,COST2,COST3,COST4,COST11,COST21,COST31,COST41
      GO TO 1000
  999 WRITE(6,10)
C
C     NEW DATA
C
 1000 GO TO 15
C
C     FORMAT STATEMENTS
C
    1 FORMAT(8I10)
    2 FORMAT(8F10.2)
    3 FORMAT(1H1,//14X,7H PERIOD,18X,5HBEGIN,18X,9HSHORTAGES,17X,
     1 6HDEMAND,21X,3HEND//)
    4 FORMAT(1H0,6X,I12,4F25.2)
    5 FORMAT(/////36X,9H CARRYING,17X,9HSHORTAGES,13X,14HREPLENISHMENTS,
     1 16X,5HTOTAL//)
    6 FORMAT(1H0,6X,7HAVERAGE,5X,3F25.2//6X,10H UNIT COST,3X,3F25.2)
    7 FORMAT(1H0,6X,11HCOST/PERIOD,1X,4F25.2/////6X,11H TOTAL COST,
     1 2X,4F25.2)
    8 FORMAT(I10,F10.2)
   10 FORMAT(1H1,///////40X,52HREPLENISHMENTS ARE NOT EQUAL TO DEMANDS.
     1CHECK DATA.)
C
      END
$DATA
        12
§4.1   10.       20.       20.       30.       20.       30.        0.        0.
       40.       30.       20.       20.
Ex. 1  .20      5.00     10.00
        0.
         1       60.
         4       60.
         7       60.
        10       60.
       -99
        12
       10.       20.       20.       30.       20.       30.        0.        0.
§4.2   40.       30.       20.       20.
       .20      5.00     10.00
Ex. 2   0.
         1      40.0
         3      40.0
         5      40.0
         7      40.0
         9      40.0
        11      40.0
       -99
        12
§4.3   10.       20.       20.       30.       20.       30.        0.        0.
Ex. 3  40.       30.       20.       20.
       .20      5.00     10.00
      10.0
```

ISL-1 (Cont'd)

```
           1      40.0
           3      40.0
           5      40.0
           7      40.0
           9      40.0
          11      40.0
         -99
-----------------------------------------------------------------------------------------
          12
§4.4     10.       20.       20.       30.       20.       30.       0.        0.
         40.       30.       20.       20.
         .20       5.00      10.00
       10.0
          11     230.0
         -99
-----------------------------------------------------------------------------------------
          12
         10.       20.       20.       30.       20.       30.       0.        0.
         40.       30.       20.       20.
§5.1     .20       5.00      10.00
          0.
Ex.4       1      30.0
           3      50.0
           5      50.0
           9      70.0
          11      40.0
         -99
-----------------------------------------------------------------------------------------
          12
§5.2     10.       20.       20.       30.       20.       30.       0.        0.
         40.       30.       20.       20.
Ex.5     .20       5.00      10.00
       10.0
           2      40.0
           4       80.0
           9      70.0
          11      50.0
         -99
-----------------------------------------------------------------------------------------
           7
§5.3    5.0       5.0       5.0       5.0       5.0      10.0      0.0
Ex.6   10.0      20.0     400.0
          0.
           1      20.0
           5      15.0
         -99
-----------------------------------------------------------------------------------------
           7
§5.4    5.0       5.0       5.0       5.0       5.0      10.0      0.0
       10.0      20.0     400.0
Ex.7      0.
           1      15.0
           5      20.0
         -99
-----------------------------------------------------------------------------------------
```

ISL-2: The Deterministic Reorder Point--Lot Size System

```
C      ISL-2 BY E.NADDOR AND RICHARD SACHER
C      FORTRAN VERSION BY RICHARD SACHER
C      THE JOHNS HOPKINS UNIVERSITY, BALTIMORE, MD 21218
C
C      BASED ON MANUAL ISL-2, DECEMBER, 1967
C
       COMMON XU, XC, XS,I1,I2,I3
       DIMENSION COSTS(3), S9(3), Q9(3)
       INTEGER X1,X2,X3,D
       REAL J1,I1,I2,I3
C
C      PRIMARY INPUT
C
   500 READ(5,1) XU,C1,C2,C3
     1 FORMAT(4F10.2)
C
C      SECONDARY INPUT
C
  1000 READ(5,4) XS,XQ,X1,XQ1,N,N1,X2,X3,J1,D
     4 FORMAT(2F10.2,I10,F10.2,4I5,F10.2,I10)
C
C      SIMULATION AND/OR AVERAGES AND/OR COSTS
C
       IF (X1) 1160,1160,2000
  1160 IF (X2) 1170,1170,3000
  1170 IF (X3) 1180,1180,4000
C
C      TO PRIMARY OR SECONDARY INPUT
C
  1180 IF (D) 500,1000,500
C
C      SIMULATES THE SYSTEM OVER A GIVEN NUMBER OF PERIODS
C
  2000 WRITE(6,2)
     2 FORMAT(1H1,16X,88HT H E   D E T E R M I N I S T I C   R E O R D E R
      1 P O I N T - L O T   S I Z E   S Y S T E M///)
       WRITE(6,3) C1,C2,C3,XU
     3 FORMAT(17H CARRYING COST = ,F6.2,20H PER UNIT PER PERIOD,39X,
      116HSHORTAGE COST = ,F6.2,20H PER UNIT PER PERIOD/21H REPLENISHING
      2COST = ,F6.2,18H PER REPLENISHMENT,37X,9HDEMAND = ,F6.2,
      3 17H UNITS PER PERIOD)
       WRITE(6,5)  XS,XQ
     5 FORMAT(//21X,17H REORDER POINT = ,F6.2,32X,11HLOT SIZE = ,F6.2//)
       WRITE(6,6)
     6 FORMAT(1H ,50X,19HS I M U L A T I O N////)
       A1=0.
       A2=0.
       A3=0.
       IF (N1) 2040,2050,2040
  2040 WRITE(6,16)
    16 FORMAT(5X,7H PERIOD,5X,5HBEGIN,5X,6HDEMAND,5X,3HEND,27X,8HCARRYING
      1,11X,9HSHORTAGES,6X,13HREPLENISHMENT/)
  2050 DO 2090  I=1,N
       IF (XQ1) 2059,2059,2052
  2052 IF (XQ1-XU) 2056,2053,2053
  2053 I1=XQ1 - XU/2.
       I2=0.
       GO TO 2061
  2056 I1 = XQ1**2 / (2.*XU)
```

```
      I2 = (XQ1-XU)**2/(2.*XU)
      GO TO 2061
 2059 I1=0.
      I2 = XU/2. - XQ1
 2061 XQ2 = XQ1 - XU
      IF (XQ2 - XS) 2063,2063,2066
 2063 I3=1.
      Y = XQ2 + XQ
      GO TO 2069
 2066 I3=0.
      Y = XQ2
 2069 IF (I - N1) 2070,2070,2080
 2070 I4 = I3
      WRITE(6,8) I,XQ1,XU,XQ2,I1,I2,I4
    8 FORMAT(/1H ,I9,F12.2,2F10.2,13X,F20.2,F19.2,I15)
 2080 XQ1 = Y
      A1 = A1 + I1
      A2 = A2 + I2
 2090 A3 = A3 + I3
C
C     COMPUTES AVERAGES FOR THE SIMULATION
C
      WRITE (6,21)  N
   21 FORMAT (/// 23H  FOR A SIMULATION OF   , I4, 9H  PERIODS//)
      XN = N
      I1 = A1/XN
      I2 = A2/XN
      I3 = A3/XN
      WRITE(6,9)
    9 FORMAT(//67X,9H CARRYING,11X,9HSHORTAGES,6X,14HREPLENISHMENTS//)
      WRITE(6,10) I1,I2,I3,C1,C2,C3
   10 FORMAT(9H AVERAGES,51X,F15.2,2X,2F17.2//11H UNIT COSTS,49X,F15.2,
     1 2X,2F17.2)
      COST1 = C1*I1
      COST2 = C2*I2
      COST3 = C3*I3
      COST4 = COST1 + COST2 + COST3
      WRITE(6,11) COST1,COST2,COST3,COST4
   11 FORMAT(/17H COSTS PER PERIOD,43X,F15.2,2X,2F17.2//25H TOTAL COSTS
     1PER PERIOD =,F8.2)
      GO TO 1160
C
C     COMPUTES THE LONG TERM AVERAGES OF THE SYSTEM
C
 3000 WRITE(6,2)
      WRITE(6,3) C1,C2,C3,XU
      WRITE(6,5) XS,XQ
      WRITE(6,17)
   17 FORMAT(/////43X,34H L O N G   T E R M   A V E R A G E S /////)
      WRITE(6,9)
      CALL AVGES
      COST1 = C1*I1
      COST2 = C2*I2
      COST3 = C3*I3
      COST4 = COST1 + COST2 + COST3
      WRITE(6,10) I1,I2,I3,C1,C2,C3
      WRITE(6,11) COST1,COST2,COST3,COST4
      GO TO 1170
C
```

```
C      COMPUTES TOTAL COSTS FOR A SPECIFIED REORDER POINT AND LOT SIZE AND A
C      NEIGHBORHOOD ABOUT THEM
C
 4000 WRITE(6,2)
      WRITE(6,3) C1,C2,C3,XU
      WRITE(6,5) XS,XQ
      WRITE(6,18)
   18 FORMAT(/////43X,37H T H E  T O T A L  C O S T  T A B L E
     1////56X,9H LOT SIZE//)
      DO 600  L = 1,3
      XL = L
      A9 = (XL-2.)*J1
      S9(L) = XS + A9
  600 Q9(L) = XQ + A9
      WRITE(6,12) (Q9(L), L = 1,3)
   12 FORMAT(1H ,23X,3F20.2)
      WRITE(6,13)
   13 FORMAT(14H REORDER POINT)
      DO 4260  I=1,3
      XS = S9(I)
      DO 4240   J=1,3
      XQ = Q9(J)
      CALL AVGES
 4240 COST5(J) = C1*I1 + C2*I2 + C3*I3
 4260 WRITE(6,14) XS,COST5
   14 FORMAT(/1H ,F9.2,14X,3F20.2)
C
C      TO PRIMARY OR SECONDARY INPUT
C
      GO TO 1180
C
      END
```

```
      SUBROUTINE AVGES
C
C      COMPUTES AVERAGES FOR THE LONG TERM AVERAGES AND THE TOTAL COSTS OPTION
C
      COMMON XU, XQ, XS,I1,I2,I3
      REAL I1,I2,I3
      I3 = XU/XQ
      IF (XS) 5050,5020,5020
 5020 I1 = XS + XQ/2.
      I2 = 0.
      RETURN
 5050 IF (XS + XQ) 5090,5060,5060
 5060 I1 = (XQ+XS)**2/(2.*XQ)
      I2 = XS**2/(2.*XQ)
      RETURN
 5090 I1 = 0.
      I2 = XQ/2. - XS
      RETURN
      END
```

```
$DATA
Ill. 3.1   5.0        1.0   99999.0        36.0
Ex. 3.11   5.0       20.0         1        15.0    10    10    0    0        0.0          0
Ex. 3.12   5.0       20.0         1        15.0   100     0    1    0        0.0          0
Ex. 3.13   5.0       20.0         0         0.0     0     0    0    1        5.0          1
Ill. 3.2   5.0   99999.0        9.0        36.0
Ex. 3.21 -10.0       10.0         1         0.0    10     4    0    0        0.0          0
Ex. 3.22 -10.0       10.0         1         0.0   100     0    1    1        2.5          1
```

ISL-2 (Cont'd)

```
Ill. 3.3    5.0        1.0        9.0       0.0
Ex. 3.11   -2.0        5.0          1       3.0     10     4     0     0      0.0          0
Ex. 3.12   -1.0        5.0          1       4.0     10     0     1     1      0.5          1
Ill. 3.4    5.0        1.0        9.0      36.0
Ex. 3.41   -1.0       20.0          1      14.0     10    10     0     0      0.0          0
Ex. 3.42   -1.0       20.0          0       0.0      0     0     1     1      1.0          1
Ex. 3.51    5.0        1.0        9.0      36.0
           -2.0       20.0          1      14.0    100    10     1     1      0.1          1
Ex. 3.52    1.0        0.2        1.8      36.0
           -2.0       20.0          1       4.0    100     8     1     1      0.1          1
         2400.0       0.56    99999.0      42.0
Ex. 3.53    0.0      600.0          1     600.0     10     5     1     1     10.0          1
          200.0   0.046667    99999.0      42.0
Ex. 3.54    0.0      600.0          1     600.0     10     5     1     1     10.0          1
```

ISL-3: The Probabilistic Reorder Point--Lot Size System

```
C     ISL-3 BY E.NADDOR AND ISRAEL PRESSMAN
C     FORTRAN VERSION BY RICHARD SACHER
C     THE JOHNS HOPKINS UNIVERSITY, BALTIMORE MD, 21218
C
C     BASED ON MANUAL ISL-3, OCTOBER, 1967
C
C     THIS PROGRAM IS FORMATTED FOR A 132 POSITION PRINTER.
C     TO CHANGE FOR A 120 POSITION PRINTER, SUBTRACT 6 SPACES FROM
C     EACH FORMAT CARD.
C
      DIMENSION P(50),F(50),Q99(50),G(50),W(50),V(50),COST(3)
      INTEGER T,S,R
      REAL N(50),IO,I1,I2,I3,J4,L1,L2,L3,L6
      COMMON C,C1,C2,C3,I1,I2,I3,Q,S1,U,N,V,X5,W1
 3020 DO 3030 J=1,50
      P(J)=0.0
 3030 Q99(J)=0.0
 3040 WRITE(6,1)
    1 FORMAT(1H1,22X,88HT H E   P R O B A B A L I S T I C   R E O R D E R
     1 P O I N T - L O T   S I Z E   S Y S T E M)
C
C     READING AND PRINTING OF COST DATA
C
 3060 READ(5,2) C1,C2,C3
    2 FORMAT(8F10.2)
 3070 WRITE(6,3) C1,C2,C3
    3 FORMAT(//6X,16H CARRYING COST =,F9.2,20H PER UNIT PER PERIOD,35X,
     1 15HSHORTAGE COST =,F9.2,20H PER UNIT PER PERIOD//43X,20H REPLENIS
     2HING COST =,F9.2,18H PER REPLENISHMENT)
 3095 IRAN = 3217
 3100 W1 = 1.
C
C     READING OF DEMAND DATA
C
 3120 READ(5,2) U,X2
 3125 J4=X2/U + .01
 3130 J2=IFIX(J4)+1
 3135 X5=0.
 3137 READ(5,2) ( P(J), J=1,J2 )
 3140 DO 3155  J=1,J2
 3150 XJ = J
 3155 X5=X5+U*P(J)*(XJ-1.)
C
C     THE EQUIVALENT DISTRIBUTION
C
 3170 J=J2
 3175 XJ = XJ - 1.
 3180 A=0.
 3190 B=P(J)/XJ
 3200 Q99(J)=A + B/2.
 3210 A=A+B
 3220 J=J-1
 3225 XJ=XJ-1.
 3230 IF (J-1) 3250,3250,3190
 3250 Q99(1) = P(1) + A/2.
C
C     PRINTING OF DEMAND DATA
C
```

```
 3260 WRITE(6,5)
    5 FORMAT(///44X,7H DEMAND,11X,11HPROBABILITY,10X,10HCUMULATIVE//)
 3270 V(1)=P(1)
 3280 G(1) = Q99(1)
 3290 W(1)=G(1)
 3300 F(1)=P(1)
 3310 N(1)=W(1)
C
C      COMPUTING G,W,F,N
C
 3320 DO 3380  J=1,49
 3330 F(J+1)=F(J)+P(J+1)
 3340 V(J+1)=V(J) + F(J+1)
 3350 G(J+1)=G(J) + Q99(J+1)
 3360 W(J+1)=W(J) + G(J+1)
 3365 IF (J-J2) 3367,3367,3380
 3367 XJ = J
 3368 XJU = (XJ-1.)*U
 3370 WRITE(6,6) XJU,P(J),F(J)
    6 FORMAT(1H ,29X,3F20.2//)
 3380 N(J+1)=N(J) + W(J+1)
C
C      MAIN PROGRAM
C
 4000 WRITE (6,98)
   98 FORMAT(1H1)
 4010 WRITE(6,7)
    7 FORMAT(///48X,35H          *********************    //)
C
C      READING OF DECISION VARIABLES, ETC (8 NUMBERS)
C      IF FIRST OF 8 NUMBERS IS LARGER
C      THAN 9000.0 THEN NEW COST AND DEMAND DATA ARE READ
C
 4040 READ(5,8) S1,Q,NY,IO,M,T,S,R
    8 FORMAT(2F10.2,I10,F10.2,4I10)
 4041 IF (S1-9000.0) 4042,4042,3020
 4042 XM = M
 4047 WRITE(6,9) S1,Q
    9 FORMAT(///21X,16H REORDER POINT =,F9.2,42X,10HLOT SIZE =,F9.2///)
C
C      SIMULATION AND/OR AVERAGES AND/OR COSTS
C
 4050 IF (NY-1) 4060,5000,4060
 4060 IF (S-1) 4070,6000,4070
 4070 IF (R-1) 4000,7000,4000
C
C      SIMULATION
C
 5000 WRITE(6,10)
   10 FORMAT(1H ,57X,19HS I M U L A T I O N////)
 5080 IF (T) 5120,5120,5100
C
C      PRINTING OF HEADINGS FOR DETAILS OF SIMULATION
C
 5100 WRITE(6,11)
   11 FORMAT(11X,7H PERIOD,5X,5HBEGIN,5X,6HDEMAND,5X,3HEND,27X,
     1 8HCARRYING,11X,9HSHORTAGES,6X,13HREPLENISHMENT)
C
C      INITIALIZE SIMULATION
C
```

ISL-3 (Cont'd)

```
 5120 B1=I0
 5130 L1 = 0.
 5140 L2=0.
 5150 L3=0.
 5155 L6=0.
 5160 K=0.
C
C      MAIN SIMULATION
C
 5170 DO 5480  J=1,M
C
C      RANDOM DEMAND X
C
 5180 Y = UDRNRT(IRAN)
 5190 J9 = 1
 5200 IF ( F(J9)-Y ) 5210,5210,5225
 5210 J9=J9+1
 5220 GO TO 5200
 5225 XJ9 = J9 - 1
 5230 X=XJ9*U
 5240 E1=B1-X
C
C      AVERAGES* CARIED(I1),SHORT(I2)
C
 5250 IF (B1) 5300,5300,5260
 5260 IF (E1) 5330,5270,5270
 5270 I1=(B1 + E1)/2.
 5280 I2=0.
 5290 GO TO 5350
 5300 I2=(-E1-E1)/2.
 5310 I1=0.
 5320 GO TO 5350
 5330 I1=B1**2/(2.*(B1-E1))
 5340 I2=E1**2/(2.*(B1-E1))
C
C      REPLENISHMENT* I3(0 OR 1)
C
 5350 IF (E1-S1) 5360,5360,5410
 5360 I3=1.
 5370 E2=E1
 5380 E2=E2 + Q
 5390 IF (E2-S1) 5380,5380,5430
 5410 I3=0.
 5420 E2=E1
 5430 IF (J-T) 5435,5435,5450
C
C      PRINTING OF DETAILS OF SIMULATION
 5435 NI3 = I3
 5440 WRITE(6,12) J,B1,X,E1,I1,I2,NI3
   12 FORMAT(1H0,6X,I9,F12.2,2F10.2,13X,F20.2,F19.2,I15)
C
C      ACCUMULATION OF AVERAGES
C
 5450 L1=L1 + I1
 5460 L2=L2 + I2
 5470 L3=L3 + I3
 5475 L6=L6 + X
 5480 B1=E2
C
C      SUMMARY OF SIMULATION AVERAGES
```

```
C
 5515 XM = M
 5520 I1=L1/XM
 5530 I2=L2/XM
 5540 I3=L3/XM
 5545 X6=L6/XM
 5549 WRITE(6,8504) M
 8504 FORMAT (///5X, 20HFOR A SIMULATION OF ,I6,8H PERIODS)
 5550 WRITE(6,8505)
 8505 FORMAT(//54X,7H DEMAND,13X,8HCARRYING,11X,9HSHORTAGES,5X,
     1 14HREPLENISHMENTS)
 5551 WRITE(6,8520) X6,I1,I2,I3,C1,C2,C3
 8520 FORMAT(/6X,9H AVERAGES,26X,2F20.2,F19.2,F17.2//6X,11H UNIT COSTS,
     1 49X,F15.2,2X,2F17.2)
 5552 COST1=C1*I1
 5553 COST2=C2*I2
 5554 COST3 = C3*I3
 5555 COST4=COST1 + COST2 + COST3
 5556 WRITE(6,8540) COST1,COST2,COST3,COST4
 8540 FORMAT(/6X,17H COSTS PER PERIOD,43X,F15.2,2X,2F17.2//6X,25H TOTAL
     1COSTS PER PERIOD =,F8.2)
      WRITE(6,99)
   99 FORMAT(/////1H )
 5560 GO TO 4060
C
C      LONG TERM AVERAGES
C
 6000 CALL AVER
 6030 WRITE(6,13)
   13 FORMAT(48X,34H L O N G   T E R M   A V E R A G E S)
 6035 X6=X5
 6040 WRITE(6,8505)
 6041 WRITE(6,8520) X6,I1,I2,I3,C1,C2,C3
 6042 COST1=C1*I1
 6043 COST2=C2*I2
      COST3=C3*I3
 6044 COST4=COST1 + COST2 + COST3
 6045 WRITE(6,8540) COST1,COST2,COST3,COST4
      WRITE(6,99)
 6070 GO TO 4070
C
C      EXPECTED TOTAL COST TABLE
C
 7000 WRITE(6,14)
   14 FORMAT(43X,47H E X P E C T E D   T O T A L   C O S T   T A B L E////
     1 62X,9H LOT SIZE)
 7035 Q11=Q-L
 7036 Q12=Q
 7037 Q13=Q + L
 7040 WRITE(6,15) Q11,Q12,Q13
   15 FORMAT(1H0,29X,3F20.2/21X,14H REORDER POINT)
 7060 S1=S1-2.*L
 7070 Q = Q + L
 7080 DO 7180  I=1,3
 7090 S1=S1 + L
 7110 Q=Q-3.*L
 7120 DO 7150  J=1,3
 7130 Q=Q + L
 7140 CALL AVER
 7150 COST(J) = C
 7180 WRITE(6,16) S1,COST
   16 FORMAT(1H0,5X,4F20.2)
 7390 GO TO 4000
C
      END
      SUBROUTINE AVER
      DIMENSION V(50)
      REAL N(50),I0,I1,I2,I3,J4,L1,L2,L3,L6
      INTEGER T,S,R
      COMMON C,C1,C2,C3,I1,I2,I3,Q,S1,U,N,V,X5,W1
 8000 B=(S1 + Q - U)/U + .01
 8005 NB = INT(B) + 1
 8010 IF (NB - 1) 8020,8050,8050
 8020 I1=0.
 8030 GO TO 8150
 8050 IF (S1) 8100,8100,8060
 8060 A = (S1 - U)/U + .01
 8069 NA = INT(A) + 1
 8080 I1=((U**2)/Q)*(N(NB)-N(NA))
 8090 GO TO 8150
 8100 I1=((U**2)/Q)*N(NB)
 8150 I2=I1 + X5/2. - ((Q+U)/2.) - S1
 8151 IF (I2 - .000001) 8153,8155,8155
 8153 I2 = 0.
 8155 D =(Q-U)/U + .01
 8160 ND = INT(D) + 1
 8200 I3 = (1.- (U/Q)*V(ND))/W1
 8210 C=C1*I1 + C2*I2 + C3*I3
      RETURN
      END
      FUNCTION UDRNRT(J)
C     THE FUNCTION UDRNRT IS A UNIFORMLY DISTRIBUTED RANDOM NUMBER GENERATOR
C     WRITTEN BY MANDELL BELLMORE, THE JOHNS HOPKINS UNIVERSITY
C     THIS FUNCTION WAS WRITTEN SPECIFICALLY FOR THE IBM 7094.  THE ARGUMENT,
C     IRAN, MUST BE INITIALIZED TO AN ODD INTEGER NOT DIVISIBLE BY 5 AND MUST
C     LIE BETWEEN 0 AND 4194303
      JRAN = J
      JRAN = JRAN*2051
      JRAN = JRAN - (JRAN/4194304)*4194304
      J = JRAN
      U = JRAN
      Y = U/4194303.0
      UDRNRT = Y
      RETURN
      END
$DATA
       5.          50.         40.
§4.1   2.           8.
      .05          .24         .38       .21        .12
Ex.4.11 -4.0       14.0          1      10.0         10        10         0         0
Ex.4.12 -4.0       14.0          1      10.0        500        10         1         1
Ex.4.13  2.0        8.0          0       0.0          0         0         0         1
Ex.4.14  0.0       10.0          1       4.0       1000         0         1         1
    9999.0          0.0          0       0.0          0         0         0         0
       1.0          9.0       36.0
§4.2   5.0          5.0
       0.0          1.0
Ex.4.21  0.0       20.0          0       0.0          0         0         0         1
    9999.0          0.0          0       0.0          0         0         0         0
```

ISL-3 (Cont'd)

Sil.	0.2 1.0 0.0	1.8 1.0 1.0	36.0					
Ex 9.22	2.0	20.0	0	0.0	0	0	0	1

```
C       ISL-4 BY E.NADDOR
C       FORTRAN VERSION BY RICHARD KESSINGER AND JON WESTON
C       THE JOHNS HOPKINS UNIVERSITY, BALTIMORE MD. 21218
C
C       BASED ON MANUAL ISL-4, OCTOBER, 1967
C
        INTEGER N4,T9,IH,B,L,L1,L3,H,X1,X2,X3,C,X,E,W,J,V,M,N,O
        INTEGER X6,X7,L4,L5,IX
        REAL C9,P9,C2,C3,C4,C1,FN4,G,F,XA,XB,X4,X5,XD,P,A,Q1,Q2,Q,R,Z
        REAL I1,I2,I3,I4,J1,J2,J3,J4,K1,K2,K3,K4,KO
C
C       SUBROUTINE RRN(I) GENERATES RANDOM NUMBERS
C       IN THE INTERVAL 0 TO 1.  SUPPLIED BY THE USER
C
        LETY=RRN(12)
C
1000    DIMENSION F(11),G(20),V(3),IH(10),R(20),IR(20)
1002    DIMENSION X(500)
C
C       READING OF DATA AND INITIALIZATION
C
1005    READ (5,1) N4
1007    IF(N4.EQ.9999)GO TO 9998
   1    FORMAT (I10)
1010    READ (5,2)T9,C9,P9,H,C2,C3,C4
   2    FORMAT (I10,F10.4,F10.4, I10,F10.4,F10.4,F10.4)
1015    FT9=T9
1020    C1 = (FT9*C9*P9)/36000.
1030    READ (5,3) B
   3    FORMAT (I10)
1040    IF (B.EQ.2) GO TO 1080
1050    READ (5,3) L
1070    GO TO 1160
1080    READ (5,4) L1,L,L3
     4  FORMAT (3I10)
        L4=L1+1
        L5=L+1
1110    DO 1130 I=L4,L5,L3
1120    READ (5,5) F(I)
   5    FORMAT (F10.4)
1130    CONTINUE
1160    READ(5,3) C
1170    IF (C.EQ.1) GO TO 1230
1180    READ (5,4) X1,X2,X3
        X6=X1+1
        X7=X2+1
1185    READ (5,5) G(X6)
1187    FN4 = N4
1190    IH(X6) = G(X6)*FN4 +.01
1193    IA = X6 + X3
1195    DO 1210 I=IA,X7,X3
1200    READ (5,5) G(I)
1203    IB = I-X3
1205    IH(I) = (G(I) -G(IB))*FN4 +.01
1210    CONTINUE
1220    GO TO 1350
1230    X4=C
1240    X5=C
1250    I=1
```

```
1260  READ(5,6)X(I)
  6   FORMAT (I10)
1270  IF (X(I).EQ.9999) GO TO 1320
1275  XD = X(I)
1280  X4 =X4 +XD
1290  X5 = X5 + XD*XD
1300  I=I+1
1310  GO TO 1260
1320  N4 = I-1
 1325 FN4=N4
1330  GO TO 1470
C
C      GENERATION OF N4 RANDOM NUMBERS
C
1350  X4  =0
1360  X5=0
1370  DO 1460 I=1,N4
1380  IX=X1+1
1390  Y=RRN(0)
1400  IF (Y.LT.G(IX)) GO TO 1430
1410  IX = IX + X3
1420  GO TO  1400
1430  IF (IH(IX).EQ.0) GO TO 1380
 1432 X(I)=IX-1
 1434 FX=X(I)
1435  IH(IX) =IH(IX) -1
1440  X4 = X4+FX
1450  X5 = X5 + FX*FX
1460  CONTINUE
C
C      MEAN AND STANDARD DEVIATION
C
1470  X5 = SQRT((X5-X4*X4/FN4)/FN4)
1480  X4 = X4/FN4
C
C      PRINTING OF HEADINGS AND DATA
C
2000  WRITE (6,7)
   7  FORMAT (/)
2010  WRITE (6,8) T9
   8  FORMAT (42H1INVENTORY SYSTEM SIMULATION, ONE PERIOD =,I10,5H DAYS)
2020  WRITE (6,7)
2030  WRITE (6,9) C9,P9,C1
   9  FORMAT(13H UNIT COST = ,F10.4,16H PERCENT/YEAR = ,F10.4,29H CARRYI
     CNG COST/UNIT/PERIOD = ,F10.4)
2040  WRITE (6,7)
2050  IF (M.EQ.2) GO TO 2080
2060  WRITE (6,10) C2
  10  FORMAT  (50H SHORTAGES MADE UP.   SHORTAGE COST/UNIT/PERIOD = ,F10
     C.4)
2070  GO TO 2090
2080  WRITE (6,11) C2
  11  FORMAT(52H SHORTAGES NOT MADE UP.   SHORTAGE COST/UNIT/PERIOD=,F10
     C.4)
2090  WRITE (6,7)
2095  WRITE (6,12) C3,C4
  12  FORMAT (22H REPLENISHING COSTS = ,F10.4,19H REVIEWING COSTS = ,F10
     C.4)
2096  WRITE (6,7)
2100  IF (B.EQ.2) GO TO 2130
```

```
2110  WRITE (6,13) L
  13  FORMAT (12H LEADTIME = ,I10,8H PERIODS)
2120  GO TO 2190
2130  WRITE (6,14)
  14  FORMAT (31H       LEADTIME     PROBABILITY)
2140  WRITE (6,7)
2150  WRITE(6,15) L1, F(L1+1)
  15  FORMAT (1H ,I10,5X,F10.4)
2155  IC = L1+L3+1
2160  DO 2180 I=IC,L5,L3
2165  ID = I-L3
2167  XA =F(I)-F(ID)
2168  IAA=I-1
2170  WRITE (6,15) IAA,XA
2180  CONTINUE
2190  WRITE (6,7)
2200  WRITE (6,7)
2205  IF (O.EQ.2) GO TO 2270
2210  WRITE (6,16)
  16  FORMAT (7H DEMAND)
2215  WRITE (6,7)
2230  DO 2250 I=1,N4
2240  WRITE (6,17) X(I)
  17  FORMAT (1H ,7I11)
2250  CONTINUE
2260  GO TO 2350
2270  WRITE (6,18)
  18  FORMAT (29H       DEMAND     PROBABILITY)
2280  WRITE (6,7)
2290  WRITE (6,19) X1,G(X6)
  19  FORMAT (1H ,I10,5X,F10.4)
2300  IG = X6+X3
2310  DO 2340 I=IG,X7,X3
2320  IJ=I-X3
2325  XB = G(I)-G(IJ)
2326  IAB=I-1
2330  WRITE(6,19) IAB,XB
2340  CONTINUE
2350  WRITE (6,7)
2360  WRITE (6,20) N4,X4,X5
  20  FORMAT (4H IN ,I10,27H PERIODS- AVERAGE DEMAND = ,F10.4,22H STANDA
     CRD DEVIATION = ,F10.4)
C
C     MAIN PROGRAM
C
3000  WRITE (6,7)
3010  WRITE (6,21)
  21  FORMAT (76H ******************************************************
     C*********************)
3020  WRITE (6,7)
3030  E=1
C
C     READING OF CONTROLLABLE VARIABLES, ETC. (8 NUMBERS)
C
3050  READ(5,23) J,V(1),V(2),V(3),M,N,O,X3
  23  FORMAT (8I10)
3051  IF(J.EQ.9999)GO TO 1005
3040  WRITE (6,22)
  22  FORMAT  (76H1    POLICY,  WHEN,    HOW MUCH, REVIEW, SIM RUN,PRINT
     CS, ADV VARIABLE, STEP )
```

```
3045  WRITE (6,7)
3055  WRITE (6,24) J,V(1),V(2),V(3),M,N,U,X3
  24  FORMAT (1H ,8I9)
C
C     INITIALIZATION OF SIMULATION
C
4000  WRITE (6,7)
4010  WRITE (6,30)
  30  FORMAT (76H -   -   -   -   -   -   -   -   -   -   -   -   -   -
     C -   -   -   -   -  )
4020  WRITE (6,7)
4030  Z=V(1)
4040  Q= V(2)
4050  W=V(3)
4500  J1=0.
4510  J2=0.
4520  J3=0.
4530  J4=0.
4540  IF (J.EQ.1) GO TO 4580
4550  Q2=Q
4570  GO TO 4610
4580  Q2=Z+Q
4610  IK=L+2
4615  DO 4630 I=1,IK
4620  R(I)=0.
4630  CONTINUE
C
C     PRINTING OF POLICY AND ITS PARAMETERS
C
5000  IF (J.EQ.2) GO TO 5040
5010  IF (J.EQ.3) GO TO 5060
5020  WRITE (6,25) Z,Q
  25  FORMAT (17H REORDER POINT = ,F10.0,12H LOT SIZE = ,F10.0)
5030  GO TO 5070
5040  WRITE (6,26)Z,Q
  26  FORMAT (17H REORDER POINT = ,F10.0,15H ORDER LEVEL = ,F10.0)
5050  GO TO 5070
5060  WRITE (6,27)Z,Q
  27  FORMAT (21H SCHEDULING PERIOD = ,F10.0,15H ORDER LEVEL = ,F10.0)
5070  WRITE (6,7)
5072  WRITE(6,28) W
  28  FORMAT (20H REVIEWING PERIOD = ,I10)
C
C     DETAILED SIMULATION HEADING
C
6000  IF (N.LE.0) GO TO 7000
6010  WRITE (6,7)
6020  WRITE (6,31)
  31  FORMAT (68H     PER    BEG    DEM    END    AVA    ORD    REC    CAR    SH
     CO   REP   REV)
6030  WRITE (6,32)
  32  FORMAT (59H                                                     ROUNDED
     C OFF)
6050  WRITE (6,7)
C
C     MAIN SIMULATION
C
 7000 MPLUSL=M+L
 7001 DO 7950 K=1,MPLUSL
C
```

```
C      Q1 AND Q2= BEGIN AND END INVENTORIES
C
 7010 Q1=Q2+R(1)
C
C      DEMAND=FX
C
 7012 FK=K
 7014 FN4=N4
 7015 INTFK=INT((FK-0.01)/FN4)
 7016 IXSUB=K-N4*INTFK
 7020 FX=X(IXSUB)
 7030 Q2=Q1-FX
C
C      AVERAGE CARRIED=I1, AVERAGE SHORT = I2
C
 7100 IF(Q2.GE.0.0) GO TO 7150
 7110 IF(Q1.GT.0.0) GO TO 7180
 7120 I1=0.
 7130 I2=-(Q1+Q2)/2.0
 7140 GO TO 7200
 7150 I1=(Q1+Q2)/2.0
 7160 I2=0.
 7170 GO TO 7200
 7180 I2=Q2**2/(2.*FX)
 7190 I1=Q1**2/(2.*FX)
 7200 IF(H.EQ.1) GO TO 7540
 7205 IF(Q2.GE.0.) GO TO 7540
 7210 I2=-Q2
 7220 Q2=0.0
C
C      A=AMOUNT ON HAND AND ON ORDER
C
 7520 A=0.
 7530 GO TO 7550
 7540 A=Q2
 7550 IF(L.EQ.0) GO TO 7590
 7560 DO 7575 I=2,L5
 7570 A=A+R(I)
 7575 CONTINUE
C
C      I3=REPLENISH(0 OR 1)I4=REVIEW (0 OR 1)
C      P=AMOUNT REPLENISHED
C
 7590 P=0.
 7600 I3=0.
 7605 I4=0.
 7610 IF(J.EQ.3) GO TO 7680
 7620 IF(K-W*(K/W).EQ.0) GO TO 7626
 7624 GO TO 7800
 7626 I4=1.
 7630 IF(A.GT.Z) GO TO 7800
 7640 IF(J.EQ.2) GO TO 7690
 7650 P=P+Q
 7660 IF(A+P.LE.Z) GO TO 7650
 7670 GO TO 7710
 7680 IZ=INT(Z+.5)
 7681 IF(K-IZ*(K/IZ).NE.0) GO TO 7800
 7685 I4=1.
 7690 P=Q-A
7692  IP=P
```

```
7700  IF(IP.EQ.0) GO TO 7800
 7710 I3=1.0
 7720 IF(B.EQ.1) GO TO 7780
 7730 I=L1+1
C
C      RANDOM LEADTIME
C
 7740 Y=RRN(0)
 7750 IF(Y.LT.F(I)) GO TO 7790
 7760 I=I+L3
 7770 GO TO 7750
C
C      UPDATE AMOUNTS ON ORDER R(I)
C
 7780 I=L+1
 7790 R(I+1)=R(I+1)+P
 7800 DO 7825 I=1.L5
 7820 R(I)=R(I+1)
 7825 CONTINUE
 7830 R(L+2)=0.
 7875 IF(K.LE.L) GO TO 7920
C
C      ACCUMULATE I1,I2,I3
C
 7880 J1=J1+I1
 7890 J2=J2+I2
 7900 J3=J3+I3
 7910 J4=J4+I4
 7920 IF(K.GT.N) GOTO 7950
C
C      DETAILS OF SIMULATION
C
 7922 IX=INT(FX+.5)
 7923 IR(1)=INT(R(1)+.5)
 7924 IP=INT(P+.5)
 7925 II1=INT(I1+.5)
 7926 II2=INT(I2+.5)
7927  IQ1=KESWES(Q1)
7928  IQ2=KESWES(Q2)
7929  IA=KESWES(A)
7930  II3=KESWES(I3)
7931  II4=KESWES(I4)
7932  WRITE(6,7940)K,IQ1,IX,IQ2,IA,IP,IR(1),II1,II2,II3,II4
 7940 FORMAT(11I6)
 7950 CONTINUE
 7999 FM=M
 8000 I1=J1/FM
 8010 I2=J2/FM
 8020 I3=J3/FM
 8030 I4=J4/FM
 8040 K1=I1*C1
 8050 K2=I2*C2
 8060 K3=I3*C3
 8070 K4=I4*C4
 8080 K0=K1+K2+K3+K4
 8090 WRITE(6,7)
 8100 WRITE(6,7)
 8110 IF(N.LT.0) GO TO 8260
C
C      SIMULATION SUMMARY
```

ISL-4 (Cont'd)

```
8120 WRITE(6,8125)  M
8125 FORMAT(1X,2CHFCR A SIMULATION OF ,I10,8HPERIODS )
8130 WRITE(6,7)
8140 WRITE(6,8145)
8145 FORMAT(14X,5CHCARRYING          SHORTAGE      REPLENISH      REVIEW)
8160 WRITE(6,7)
8170 WRITE(6,8175) I1,I2,I3,I4
8175 FORMAT(1X,13HAVERAGES        ,F12.4,F12.4,F12.4,F12.4)
8190 WRITE(6,8195) C1,C2,C3,C4
8195 FORMAT(1X,13HUNIT COSTS      ,F12.4,F12.4,F12.4,F12.4)
8210 WRITE(6,8215) K1,K2,K3,K4
8215 FORMAT(1X,13HCCSTS           ,F12.4,F12.4,F12.4,F12.4)
8230 WRITE(6,7)
8240 WRITE(6,8245) KC
8245 FORMAT(1X,22HTCTAL COST PER PERIOD=,F12.4)
8250 GC TC 829C
C
C     BRIEF SIMULATICN SUMMARY
C
8260 WRITE(6,8265) K1,K2,K3,K4,KO
8265 FORMAT(1X,4HCAR=,F1C.4,4HSHO=,F10.4,4HREP=,F10.4,4HREV=,F10.4,
    C4HTCT=,F1C.4)
C
C     ADVANCING A VARIABLE
C
8290 IF(C.EC.0) GC TC 3000
8300 IF(E.EC.3) GCT C 3000
8310 E=E+1
8320 V(C)=V(C)+X3
8330 GO TC 4CCC
9998 STCP
9999 ENC
     FUNCTICN KESWES(X)
     IF(X.GE.C.C) GC TO 30
  40 KESWES=INT(X-.5)
     RETURN
  30 KESWES=INT(X+.5)
     RETURN
     ENC                                                           RRN*0000
$IBLCR RRN*                                                        RRN*0001
$TEXT  RRN*                                                        RRN*0002

$CCICT RRN*                                                        RRN*0004
                                                                   RRN*0005
$DKENC RRN*
$DATA
     1CC
Ill. 4.1  7  1285.714       20.        1        50.       40.       0.0
     1
     C
     2
     C          8          2
  .05C
  .29C
   .67
   .88
    1.
Ex 4.11  1      -2         8         1         5         5         0        C
Ex. 4.12 2       0        1C         1       100         8         0        (
```

ISL-4 (Cont'd)

```
Ex. 4.13    3           3          12           2         600           7           0           0
         9999        9999        9999        9999        9999        9999        9999        9999
           10
            7    1285.714         20.           2         50.         40.         1.5
III.4.2     2
            0           2           1
          .20
          .80
          1.0
            1
            4
            2
            6
            4
            8
            2
            4
         9999
Ex. 4.21    1           4           8           1          15          15           0           0
Ex. 4.22    2           4          12           2         700           0           0           0
Ex 4.23     3           2          12           1          70          -1           0           0
         9999        9999        9999        9999        9999        9999        9999        9999
          100
            7    1285.714         20.           1         50.         40.         0.0
III.5       1
            0
            2
            0           8           2
         .050
         .290
          .67
          .88
           1.
Ex. 5.1     1           0           8           1         500          -1           1           2
Ex. 5.2     1           2           6           1         500          -1           2           2
Ex. 5.3     1           2           8           1         500          -1           3           1
Ex 5.4      1           0          10           1         500          -1           0           0
         9999        9999        9999        9999        9999        9999        9999        9999
```